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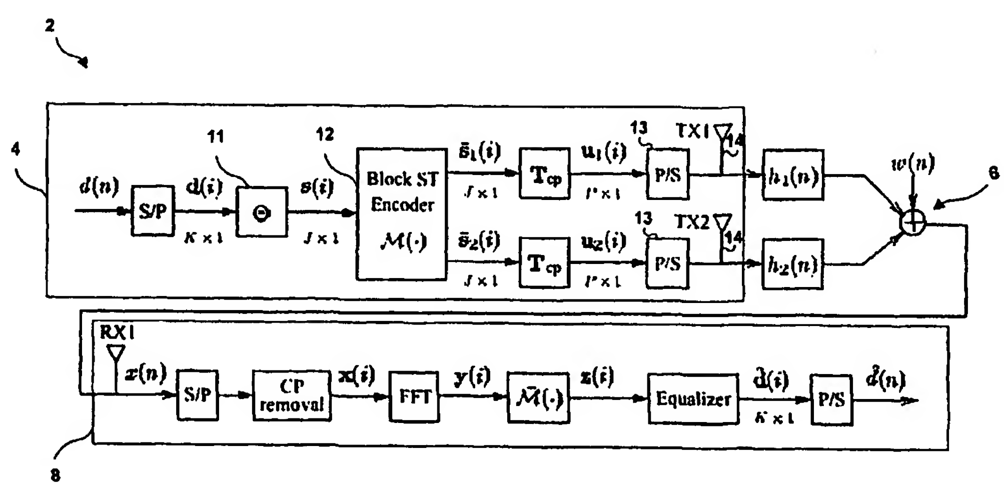

(57) Abstract: Techniques are described for space-time block coding for single-carrier block transmissions over frequency selective multipath fading channels Techniques are described that achieve a maximum diversity of order $N_tN_r(L + 1)$ in rich scattering environments, where $N_t(N_r)$ is the number of transmit (receive) antennas, and L is the order of the finite impulse responde (FIR) channels. The techniques may include parsing a stream of information-bearing symbols to form blocks of K symbols, precoding the symbols to form blocks having J symbols, and collecting consecutive N_s blocks, generating a space-time block coded matrix having N_t rows that are communicated through a wireless communication medium. The receiver complexity is comparable to single antenna transmissions, and the exact Viterbi's algorithm can be applied for maximum-likelihood (ML) optimal decoding.

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SPACE-TIME CODED TRANSMISSIONS WITHIN A WIRELESS COMMUNICATION NETWORK

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TECHNICAL FIELD

The invention relates to communication systems and, more particularly, multiple-antennae transmitters and receivers for use in wireless communication systems.

BACKGROUND

Space-time (ST) coding using multiple transmit-antennae has been recognized as an attractive means of achieving high data rate transmissions with diversity and coding gains in wireless applications. However, ST codes are typically designed for frequency flat channels. Future broadband wireless systems will likely communicate symbols with duration smaller than the channel delay spread, which gives rise to frequency selective propagation effects. When targeting broadband wireless applications, it is important to design ST codes in the presence of frequency selective multipath channels. Unlike flat fading channels, optimal design of ST codes for dispersive multipath channels is complex because signals from different antennas are mixed not only in space, but also in time. In order to maintain decoding simplicity and take advantage of existing ST coding designs for flat fading channels, most conventional techniques have pursued two-step approaches. In particular, the techniques mitigate intersymbol interference (ISI) by converting frequency selective fading channels to flat fading channels, and then design ST coders and decoders for the resulting flat fading channels. One approach to ISI mitigation has been to employ a relatively complex multiple-input

multiple-output equalizer (MIMO-EQ) at the receiver to turn FIR channels into temporal ISI-free ones.

Another approach, with lower receiver complexity, is to employ orthogonal frequency division multiplexing (OFDM), which converts frequency selective multipath channels into a set of flat fading subchannels through inverse Fast Fourier Transform (FFT) and cyclic prefix (CP) insertion at the transmitter, together with CP removal and FFT processing at the receiver. On the flat fading OFDM subchannels, many techniques have applied ST coding for transmissions over frequency-selective channels. Some of these assume channel knowledge, while others require no channel knowledge at the transmitter.

Although using ST codes designed for flat fading channels can at least achieve full multi-antenna diversity, the potential diversity gains embedded in multipath propagation have not been addressed thoroughly. OFDM based systems are able to achieve both multi-antenna and multipath diversity gains of order equal to the product of the number of transmit-antennas, the number of receive-antennas, and the number of FIR channel taps. However, code designs that guarantee full exploitation of the embedded diversity have not been explored. A simple design achieves full diversity, but it is essentially a repeated transmission, which decreases the transmission rate considerably. On the other hand, for single antenna transmissions, it has been shown that a diversity order equal to the number of FIR taps is achievable when OFDM transmissions are linearly precoded across subcarriers. An inherent limitation of multicarrier (OFDM) based ST transmissions is a non-constant modulus, which necessitates power amplifier back-off, and thus reduces power efficiency. In addition, multi-carrier schemes are more sensitive to carrier frequency offsets relative to their single-carrier counterparts.

SUMMARY

In general, the invention is directed to space-time block coding techniques for single carrier block transmissions in the presence of frequency-selective fading channels. Furthermore, in accordance with the techniques, a maximum diversity up to order $N_t N_r (L + 1)$ can be achieved in a rich scattering environment, where N_t is the number of transmit antennas, N_r is the number of receive antennas, and (L +

1) is the number of taps corresponding to each FIR channel. The techniques enable simple linear processing to collect full antenna diversity, and incur receiver complexity that is comparable to single antenna transmissions. Notably, the transmissions enable exact application of Viterbi's algorithm for maximum-likelihood (ML) optimal decoding, in addition to various reduced-complexity sub-optimal equalization alternatives. When the transmissions are combined with channel coding, they facilitate application of iterative (turbo) equalizers. Simulation results demonstrate that joint exploitation of space-multipath diversity leads to significantly improved performance in the presence of frequency selective multipath channels.

In one embodiment, a method may comprise applying a permutation matrix to blocks of symbols of an outbound data stream, and generating transmission signals from the permutated blocks of symbols. The method may further comprise communicating the transmission signals through a wireless communication medium.

In another embodiment, a method may comprise parsing a stream of information-bearing symbols to form blocks of K symbols, precoding the symbols to form blocks having J symbols, and collecting consecutive N_s blocks. The method may further comprise applying a permutation matrix to the N_s blocks, generating a space-time block coded matrix having N_t rows, each row containing N_d*J symbols, generating N_t transmission signals from the symbols of the N_t rows, and communicating the N_t transmission signals through a wireless communication medium.

In another embodiment, a transmitting device may comprise an encoder to apply a permutation matrix to blocks of information bearing symbols and to generate a space-time block coded matrix of the permutated blocks of symbols. The transmitting device further comprises a plurality of pulse shaping units to generate a plurality of transmission signals from the symbols of the space-time block coded matrix, and a plurality of antennae to communicate the transmission signals through a wireless communication medium.

The details of one or more embodiments of the invention are set forth in the accompanying drawings and the description below. Other features, objects, and

advantages of the invention will be apparent from the description and drawings, and from the claims.

BRIEF DESCRIPTION OF DRAWINGS

FIG. 1 is a block diagram illustrating a wireless communication system in which a transmitter communicates with a receiver through a wireless channel using space-time coded transmissions.

FIGS. 2 - 3 are timing diagrams illustrating the transmitted sequences from the antennas of the transmitter of FIG. 1.

FIG. 4 is another example transmission format for the transmitter of FIG. 1.

FIG. 5 is an example communication system using channel coding with the space-time coded transmission techniques in accordance with the principles of the invention.

FIGS. 6 - 9 are graphs illustrating simulated performance results for systems with two transmit and one receive antenna.

DETAILED DESCRIPTION

The Detailed Description is organized as follows: Section I deals with the special case in which a system includes of a receiver having a single antenna, and transmitter having two transmit antennas. Section II details the equalization and decoding designs. Section III generalizes the proposed schemes to multiple transmit- and receive- antennas. Simulation results are presented in Section IV.

Throughout the Detailed Description, bold upper letters denote matrices, bold lower letters stand for column vectors; $(\bullet)^*$, $(\bullet)^T$ and $(\bullet)^H$ denote conjugate, transpose, and Hermitian transpose, respectively; $E\{\bullet\}$ for expectation, $\mathrm{tr}\{\bullet\}$ for the trace of a matrix, $\|\bullet\|$ for the Euclidean norm of a vector; $\mathbf{I}_K$ denotes the identity matrix of size K, $\mathbf{0}_{M\times N}$ ($\mathbf{1}_{M\times N}$) denotes an all-zero (all-one) matrix with size MxN,and $\mathbf{F}_N$denotes an NXN FFT matrix with the (p+1; q+1)st entry of:

$$(1/\sqrt{N})\exp(-j2\pi pq/N), \forall p,q \in [0, N-1];$$

diag(x) stands for a diagonal matrix with x on its diagonal. $[\bullet]_p$ denotes the (p+1)st entry of a vector, and $[\bullet]_{p,q}$ denotes the (p+1; q+1)st entry of a matrix.

I. SINGLE CARRIER BLOCK TRANSMISSIONS

FIG. 1 is a block diagram illustrating a wireless communication system 2 in which a transmitter 4 communicates with a receiver 6 through a wireless communication channel 8. In particular, FIG. 1 illustrates the discrete-time equivalent baseband model in which transmitter 4 transmits a data with two transmit antennas ($N_t = 2$), and receiver 6 receives data with a single receive antenna ($N_r = 1$). Transmitter 4 includes a precoder 11, an encoder 12, two pulse shaping units 13 for generating transmission signals, and two transmission antennae 14.

The information-bearing data symbols d(n) belonging to an alphabet A are first parsed to K x 1 blocks $\mathbf{d}(i) := [d(iK); \ldots ; d(iK + K-1)]^T$, where the serial index n is related to the block index i by: $n = iK + k$; $k \in [0;K - 1]$. The blocks $\mathbf{d}(i)$ are precoded by a J x K matrix Θ (with entries in the complex field) to yield J x 1 symbol blocks: $s(i) := \Theta d(i)$. The linear precoding by Θ can be either non-redundant with J = K, or, redundant when J > K. The ST encoder takes as input two consecutive blocks s(2i) and s(2i + 1) to output the following 2J x 2 space-time block coded matrix:

$$\begin{bmatrix} \bar{s}_1(2i) & \bar{s}_1(2i+1) \\ \bar{s}_2(2i) & \bar{s}_2(2i+1) \end{bmatrix} := \begin{bmatrix} s(2i) & -\mathbf{P}s^*(2i+1) \\ s(2i+1) & \mathbf{P}s^*(2i) \end{bmatrix} \begin{array}{l} \to \text{time} \\ \downarrow \text{space} \end{array}, \quad (1)$$

where P is a permutation matrix that is drawn from a set of permutation matrices $\{P_J^{(n)}\}_{n=0}^{J-1}$, with J denoting the dimensionality J x J. Each performs a reverse cyclic shift (that depends on n) when applied to a J x 1 vector $\mathbf{a} := [a(0); a(1); \ldots ; a(J-1)]^T$. Specifically, $[P_J^{(n)}\mathbf{a}]_p = a((J-p+n) \bmod J)$. Two special cases are $P_J^{(0)}$ and $P_J^{(1)}$. The output of $P_J^{(0)}\mathbf{a} = [a(J - 1); a(J - 2); \ldots; a(0)]^T$ performs *time-reversal* of $\mathbf{a}$, while $P_J^{(1)}\mathbf{a} = [a(0); a(J - 1); a(J - 2); \ldots ; a(1)]^T = F_J^{(-1)} F_J^{(H)} = F_J^{(H)} F_J^{(H)}\mathbf{a}$ corresponds to taking the J-point IFFT twice on the vector $\mathbf{a}$. This double IFFT operation in the ST coded matrix is in fact a special case of a Z-transform approach originally proposed in Z. Liu and G. B. Giannakis, "Space-time coding with transmit antennas for multiple access regardless of frequency-selective multi-path,"

in Proc. of Sensor Array and Multichannel Signal Processing Workshop, Boston, MA, Mar. 2000, pp. 178–182., with the Z-domain points chosen to be equally spaced on the unit circle: $\{e^{j\frac{2\pi}{J}n}\}_{n=0}^{J-1}$. The techniques herein allow for any **P** from the set $\{\mathbf{P}_J^{(n)}\}_{n=0}^{J-1}$.

At each block transmission time interval i, the blocks $s_1(i)$ and $s_2(i)$ are forwarded to the first and the second antennae of transmitter 4, respectively. From equation (1), we have:

$$\bar{\mathbf{s}}_1(2i+1) = -\mathbf{P}\bar{\mathbf{s}}_2^*(2i), \qquad \bar{\mathbf{s}}_2(2i+1) = \mathbf{P}\bar{\mathbf{s}}_1^*(2i), \quad (2)$$

which shows that each transmitted block from one antenna at time slot 2i+1 is a conjugated and permuted version of the corresponding transmitted block from the other antenna at time slot 2i (with a possible sign change). For flat fading channels, symbol blocking is unnecessary, i.e., J = K = 1 and P = 1, and the design of (1) reduces to the Alamouti ST code matrix. However, for frequency selective multipath channels, the permutation matrix **P** is necessary as will be clarified soon.

To avoid inter-block interference in the presence of frequency selective multipath channels, transmitter 4 insert a cyclic prefix for each block before transmission. Mathematically, at each antenna $\mu \in [1, 2]$, a tall P X J transmit-matrix $\mathbf{T}_{cp} := [\mathbf{I}_{cp}^T, \mathbf{I}_J^T]^T$, with I_{cp} comprising the last P -J rows of I_J, is applied on $\bar{\mathbf{s}}_\mu(i)$ to obtain P x 1 blocks: $\mathbf{u}_\mu(i) = \mathbf{T}_{cp}\bar{\mathbf{s}}_\mu(i)$. Indeed, multiplying T_{cp} with $\bar{\mathbf{s}}_\mu(i)$ replicates the last P - L entries of $\bar{\mathbf{s}}_\mu(i)$ and places them on its top. The transmitted sequences from both antennas of transmitter 4 are depicted in FIG. 2.

With symbol rate sampling, $\mathbf{h}_\mu := [h_\mu(0); \ldots ; \mathbf{h}_\mu(L)]^T$ be the equivalent discrete-time channel impulse response (that includes transmit-receive filters as well as multipath effects) between the μth transmit antenna and the single receive antenna, where L is the channel order. With the CP length at least as long as the channel order, P - J = L, the inter block interference (IBI) can be avoided at the receiver by discarding the received samples corresponding to the cyclic prefix. CP insertion at the transmitter together with CP removal at the receiver yields the following channel input-output relationship in matrix-vector form: x(i)

$$\mathbf{x}(i) = \sum_{\mu=1}^{2} \tilde{\mathbf{H}}_{\mu}\bar{\mathbf{s}}_{\mu}(i) + \mathbf{w}(i), \tag{3}$$

where the channel matrix $\tilde{\mathbf{H}}_{\mu}$ is circulant with

$[\tilde{\mathbf{H}}_{\mu}]_{p,q} = h_{\mu}((p-q) \bmod J)$, and the additive Gaussian noise **w**(i) is assumed to be white with each entry having variance $\sigma_w^2 = N_0$.

Receiver 6 can exploit the following two properties of circulant matrices:

p1) Circulant matrices can be diagonalized by FFT operations

$$\tilde{\mathbf{H}}_{\mu} = \mathbf{F}_J^{\mathcal{H}}\mathbf{D}(\tilde{\mathbf{h}}_{\mu})\mathbf{F}_J \quad \text{and} \quad \tilde{\mathbf{H}}_{\mu}^{\mathcal{H}} = \mathbf{F}_J^{\mathcal{H}}\mathbf{D}(\tilde{\mathbf{h}}_{\mu}^{*})\mathbf{F}_J, \tag{4}$$

where

$\mathbf{D}(\tilde{\mathbf{h}}_{\mu}) := \mathrm{diag}(\tilde{\mathbf{h}}_{\mu})$, and $\tilde{\mathbf{h}}_{\mu} := [H_{\mu}(e^{j0}), H_{\mu}(e^{j\frac{2\pi}{J}}), \ldots, H_{\mu}(e^{j\frac{2\pi}{J}(J-1)})]^T$ with the pth entry being the channel frequency response

$H_{\mu}(z) := \sum_{l=0}^{L} h_{\mu}(l) z^{-l}$ evaluated at the frequency

$z = e^{j\frac{2\pi}{J}(p-1)}$.

p2) Pre- and post- multiplying $\tilde{\mathbf{H}}_{\mu}$ by **P** yields $\tilde{\mathbf{H}}_{\mu}^{T}$:

$$\mathbf{P}\tilde{\mathbf{H}}_{\mu}\mathbf{P} = \tilde{\mathbf{H}}_{\mu}^{T} \quad \text{and} \quad \mathbf{P}\tilde{\mathbf{H}}_{\mu}^{*}\mathbf{P} = \tilde{\mathbf{H}}_{\mu}^{\mathcal{H}}. \tag{5}$$

With the ST coded blocks satisfying (2), let us consider two consecutive received blocks [c.f. (3)]:

$$\mathbf{x}(2i) = \mathbf{H}_1\bar{\mathbf{s}}_1(2i) + \mathbf{H}_2\bar{\mathbf{s}}_2(2i) + \mathbf{w}(2i), \tag{6}$$

$$\mathbf{x}(2i+1) = -\tilde{\mathbf{H}}_1\mathbf{P}\bar{\mathbf{s}}_2^{*}(2i) + \tilde{\mathbf{H}}_2\mathbf{P}\bar{\mathbf{s}}_1^{*}(2i) + \mathbf{w}(2i+1). \tag{7}$$

Left-multiplying (7) by **P**, conjugating, and using p2), we arrive at:

$$\mathbf{P}\mathbf{x}^{*}(2i+1) = -\tilde{\mathbf{H}}_1^{\mathcal{H}}\bar{\mathbf{s}}_2(2i) + \tilde{\mathbf{H}}_2^{\mathcal{H}}\bar{\mathbf{s}}_1(2i) + \mathbf{P}\mathbf{w}^{*}(2i+1). \tag{8}$$

Notice that having permutation matrix P inserted at the transmitter allows the Hermitian of the channel matrices in (8) for enabling multi-antenna diversity gains with linear receiver processing.

We will pursue frequency-domain processing of the received blocks, which we describe by multiplying the blocks $\mathbf{x}(i)$ with the FFT matrix $\mathbf{F}_J$ that implements the J-point FFT of the entries in $\mathbf{x}(i)$. Let us define $\mathbf{y}(2i) := \mathbf{F}_J\mathbf{x}(2i)$, $\mathbf{y}^*(2i+1) := \mathbf{F}_J\mathbf{P}\mathbf{x}^*(2i+1)$, and likewise $\bar{\boldsymbol{\eta}}(2i) := \mathbf{F}_J\mathbf{w}(2i)$ and $\bar{\boldsymbol{\eta}}^*(2i+1) := \mathbf{F}_J\mathbf{P}\mathbf{w}^*(2i+1)$. For notational convenience, we also define the diagonal matrices $\mathcal{D}_1 := \mathrm{D}(\tilde{\mathbf{h}}_1)$ and $\mathcal{D}_2 := \mathrm{D}(\tilde{\mathbf{h}}_2)$ with the corresponding transfer function FFT samples on their diagonals. Applying the property p1) on (6) and (8), we obtain the FFT processed blocks as:

$$\mathbf{y}(2i) = \mathcal{D}_1\mathbf{F}_J\bar{\mathbf{s}}_1(2i) + \mathcal{D}_2\mathbf{F}_J\bar{\mathbf{s}}_2(2i) + \bar{\boldsymbol{\eta}}(2i), \tag{9}$$

$$\mathbf{y}^*(2i+1) = -\mathcal{D}_1^*\mathbf{F}_J\bar{\mathbf{s}}_2(2i) + \mathcal{D}_2^*\mathbf{F}_J\bar{\mathbf{s}}_1(2i) + \bar{\boldsymbol{\eta}}^*(2i+1). \tag{10}$$

It is important to remark at this point that permutation, conjugation, and FFT operations on the received blocks $\mathbf{x}(i)$ do not introduce any information loss, or color the additive noises in (9) and (10) that remain white. It is thus sufficient to rely only on the FFT processed blocks $\mathbf{y}(2i)$ and $\mathbf{y}^*(2i+1)$ when performing symbol detection.

After defining $\check{\mathbf{y}}(i) := [\mathbf{y}^T(2i), \mathbf{y}^{\mathcal{H}}(2i+1)]^T$, we can combine (9) and (10) into a single block matrix-vector form to obtain:

$$\check{\mathbf{y}}(i) = \underbrace{\begin{bmatrix} \mathcal{D}_1 & \mathcal{D}_2 \\ \mathcal{D}_2^* & -\mathcal{D}_1^* \end{bmatrix}}_{:=\mathcal{D}} \begin{bmatrix} \mathbf{F}_J\mathbf{s}(2i) \\ \mathbf{F}_J\mathbf{s}(2i+1) \end{bmatrix} + \begin{bmatrix} \bar{\boldsymbol{\eta}}(2i) \\ \bar{\boldsymbol{\eta}}^*(2i+1) \end{bmatrix}, \tag{11}$$

where the identities $\bar{\mathbf{s}}_1(2i) = \mathbf{s}(2i)$ and $\bar{\mathbf{s}}_2(2i) = \mathbf{s}(2i+1)$ have been used following our design in (1).

Consider a $J \times J$ diagonal matrix $\bar{\mathcal{D}}_{12}$ with non-negative diagonal entries as: $\bar{\mathcal{D}}_{12} = [\mathcal{D}_1^*\mathcal{D}_1 + \mathcal{D}_2^*\mathcal{D}_2]^{1/2}$. We can verify that the matrix $\mathcal{D}$ in (11) satisfies $\mathcal{D}^{\mathcal{H}}\mathcal{D} = \mathrm{I}_2 \otimes \bar{\mathcal{D}}_{12}^2$, where $\otimes$ stands for Kronecker product. Based on $\mathcal{D}_1$ and $\mathcal{D}_2$, we next construct a unitary matrix $\mathbf{U}$. If $\mathbf{h}_1$ and $\mathbf{h}_2$ do not share common zeros on the FFT grid $\{e^{j\frac{2\pi}{J}n}\}_{n=0}^{J-1}$, then $\bar{\mathcal{D}}_{12}$ is invertible, and we select $\mathbf{U}$ as $\mathbf{U} := \mathcal{D}(\mathrm{I}_2 \otimes \bar{\mathcal{D}}_{12}^{-1})$. If $\mathbf{h}_1$ and $\mathbf{h}_2$ happen to share common zero(s) on the FFT grid (although this event has probability zero), then we construct $\mathbf{U}$ as follows. Supposing without of loss of generality that $\mathbf{h}_1$ and $\mathbf{h}_2$ share a common zero at the first subcarrier e^{j0}, we have that $[\mathcal{D}_1]_{1,1} = [\mathcal{D}_2]_{1,1} = [\bar{\mathcal{D}}_{12}]_{1,1} = 0$. We then construct a diagonal matrix $\mathcal{D}_1'$ which differs from $\mathcal{D}_1$ only at the first diagonal entry: $[\mathcal{D}_1']_{1,1} = 1$. Similar to the definition of $\mathcal{D}$ and $\bar{\mathcal{D}}_{12}$, we construct $\mathcal{D}'$ and $\bar{\mathcal{D}}_{12}'$ by substituting $\mathcal{D}_1$ with $\mathcal{D}_1'$. Because $\bar{\mathcal{D}}_{12}'$ is invertible, we form $\mathbf{U} := \mathcal{D}'[\mathrm{I}_2 \otimes (\bar{\mathcal{D}}_{12}')^{-1}]$. In summary, no matter whether $\bar{\mathcal{D}}_{12}$ is invertible or not, we can always construct a unitary $\mathbf{U}$, which satisfies $\mathbf{U}^{\mathcal{H}}\mathbf{U} = \mathrm{I}_{2J}$ and $\mathbf{U}^{\mathcal{H}}\mathcal{D} = \mathrm{I}_2 \otimes \bar{\mathcal{D}}_{12}$, where the latter can be easily verified. As multiplying by unitary matrices does not incur any loss of decoding optimality in the presence of additive white Gaussian noise, (11) yields $\check{\mathbf{z}}(i) := [\mathbf{z}^T(2i), \mathbf{z}^T(2i+1)]^T$ as:

$$\check{\mathbf{z}}(i) = \mathbf{U}^{\mathcal{H}}\check{\mathbf{y}}(i) = \begin{bmatrix} \bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{s}(2i) \\ \bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{s}(2i+1) \end{bmatrix} + \mathbf{U}^{\mathcal{H}} \begin{bmatrix} \bar{\boldsymbol{\eta}}(2i) \\ \bar{\boldsymbol{\eta}}^*(2i+1) \end{bmatrix}, \tag{12}$$

where the resulting noise $\check{\boldsymbol{\eta}}(i) := [\boldsymbol{\eta}^T(2i), \boldsymbol{\eta}^T(2i+1)]^T = \mathbf{U}^{\mathcal{H}}[\bar{\boldsymbol{\eta}}^T(2i), \bar{\boldsymbol{\eta}}^{\mathcal{H}}(2i+1)]^T$ is still white with each entry having variance N_0.

We infer from (12) that the blocks $\mathbf{s}(2i)$ and $\mathbf{s}(2i+1)$ can be demodulated separately without compromising the ML optimality, after linear receiver processing. Indeed, so far we applied at the receiver three linear unitary operations after the CP removal: i) permutation (via $\mathbf{P}$); ii) conjugation and FFT (via $\mathbf{F}_J$); and iii) unitary combining (via $\mathbf{U}^{\mathcal{H}}$). As a result, we only need to demodulate each information block $\mathbf{d}(i)$ separately from the following sub-blocks [c.f. (12)]:

$$\mathbf{z}(i) = \bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{s}(i) + \boldsymbol{\eta}(i) = \bar{\mathcal{D}}_{12}\mathbf{F}_J\Theta\mathbf{d}(i) + \boldsymbol{\eta}(i). \tag{13}$$

A. Diversity gain analysis

Let us drop the block index i from (13), and e.g., use $\mathbf{d}$ to denote $\mathbf{d}(i)$ for notational brevity. With perfect CSI at the receiver, we will consider the pairwise error probability (PEP) $P(\mathbf{d} \to \mathbf{d}'|\mathbf{h}_1, \mathbf{h}_2)$ that the symbol block $\mathbf{d}$ is transmitted, but is erroneously decoded as $\mathbf{d}' \neq \mathbf{d}$. The PEP can be approximated using the Chernoff bound as

$$P(\mathbf{s} \to \mathbf{s}'|\mathbf{h}_1, \mathbf{h}_2) \leq \exp(-d^2(\mathbf{z}, \mathbf{z}')/4N_0), \tag{14}$$

where $d(\mathbf{z}, \mathbf{z}')$ denotes the Euclidean distance between $\mathbf{z}$ and $\mathbf{z}'$.

Define the error vector as $\mathbf{e} := \mathbf{d} - \mathbf{d}'$, and a $J \times (L+1)$ Vandermonde matrix $\mathbf{V}$ with $[\mathbf{V}]_{p,q} = \exp(-j2\pi pq/J)$. The matrix $\mathbf{V}$ links the channel frequency response with the time-domain channel taps as $\tilde{\mathbf{h}}_\mu = \mathbf{V}\mathbf{h}_\mu$. Starting with (13), we then express the distance as:

$$\begin{aligned} d^2(\mathbf{z}, \mathbf{z}') &= \|\bar{\mathcal{D}}_{12}\mathbf{F}_J\Theta\mathbf{e}\|^2 = \mathbf{e}^{\mathcal{H}}\Theta^{\mathcal{H}}\mathbf{F}_J^{\mathcal{H}}\bar{\mathcal{D}}_{12}^2\mathbf{F}_J\Theta\mathbf{e} \\ &= \sum_{\mu=1}^{2}\|\mathcal{D}_\mu\mathbf{F}_J\Theta\mathbf{e}\|^2 = \sum_{\mu=1}^{2}\|\mathbf{D}_e\mathbf{V}\mathbf{h}_\mu\|^2, \end{aligned} \tag{15}$$

where $\mathbf{D}_e := \mathrm{diag}(\mathbf{F}_J\Theta\mathbf{e})$ such that $\mathcal{D}_\mu\mathbf{F}_J\Theta\mathbf{e} = \mathbf{D}_e\tilde{\mathbf{h}}_\mu = \mathbf{D}_e\mathbf{V}\mathbf{h}_\mu$.

We focus on block quasi static channels, i.e., channels that remain invariant over each space-time coded block, but may vary from one block to the next. We further adopt the following assumption:

as0) *the channels $\mathbf{h}_1$ and $\mathbf{h}_2$ are uncorrelated; and for each antenna $\mu \in [1, 2]$, the channel $\mathbf{h}_\mu$ is zero-mean, complex Gaussian distributed, with covariance matrix $\mathbf{R}_{h,\mu} := E\{\mathbf{h}_\mu\mathbf{h}_\mu^{\mathcal{H}}\}$.*

If the entries of $\mathbf{h}_\mu$ are i.i.d., then we have $\mathbf{R}_{h,\mu} = \mathbf{I}_{L+1}/(L+1)$, where the channel covariance matrix is normalized to have unit energy; i.e., $\mathrm{tr}\{\mathbf{R}_{h,\mu}\} = 1$. Because general frequency selective multipath channels have covariance matrices with arbitrary rank, we define the "effective channel order" as: $\tilde{L}_\mu = \mathrm{rank}(\mathbf{R}_{h,\mu}) - 1$. Consider now the following eigen decomposition:

$$\mathbf{R}_{h,\mu} = \mathbf{U}_{h,\mu}\Lambda_{h,\mu}\mathbf{U}_{h,\mu}^{\mathcal{H}}, \tag{16}$$

where $\Lambda_{h,\mu}$ is an $(\tilde{L}_\mu+1)\times(\tilde{L}_\mu+1)$ diagonal matrix with the positive eigenvalues of $\mathbf{R}_{h,\mu}$ on its diagonal, and $\mathbf{U}_{h,\mu}$ is an $(L+1) \times (\tilde{L}_\mu + 1)$ matrix having orthonormal columns: $\mathbf{U}_{h,\mu}^{\mathcal{H}}\mathbf{U}_{h,\mu} = \mathbf{I}_{\tilde{L}_\mu+1}$. Defining $\bar{\mathbf{h}}_\mu = \Lambda_{h,\mu}^{-\frac{1}{2}}\mathbf{U}_{h,\mu}^{\mathcal{H}}\mathbf{h}_\mu$, we can verify that the entries of $\bar{\mathbf{h}}_\mu$ are i.i.d. with unit variance. Since $\mathbf{h}_\mu$ and

$\mathbf{U}_{h,\mu}\Lambda_{h,\mu}^{\frac{1}{2}}\bar{\mathbf{h}}_\mu$ have identical distributions, we replace the former by the latter in the ensuing PEP analysis. A special case of interest corresponds to transmissions experiencing channels with full rank correlation matrices; i.e., $\text{rank}(\mathbf{R}_{h,\mu}) = L+1$ and $\tilde{L}_\mu = L$. As will be clear later on, a rich scattering environment leads to $\mathbf{R}_{h,\mu}$'s with full rank, which is favorable in broadband wireless applications because it is also rich in diversity.

With the aid of the whitened and normalized channel vector $\bar{\mathbf{h}}_\mu$, we can simplify (15) to:

$$d^2(\mathbf{z},\mathbf{z}') = \|\mathbf{D}_e\mathbf{V}\mathbf{U}_{h,1}\Lambda_{h,1}^{\frac{1}{2}}\bar{\mathbf{h}}_1\|^2 + \|\mathbf{D}_e\mathbf{V}\mathbf{U}_{h,2}\Lambda_{h,2}^{\frac{1}{2}}\bar{\mathbf{h}}_2\|^2. \tag{17}$$

From the spectral decomposition of the matrix $\mathbf{A}_{e,\mu}^{\mathcal{H}}\mathbf{A}_{e,\mu}$, where $\mathbf{A}_{e,\mu} := \mathbf{D}_e\mathbf{V}\mathbf{U}_{h,\mu}\Lambda_{h,\mu}^{\frac{1}{2}}$, we know that there exists a unitary matrix $\mathbf{U}_{e,\mu}$ such that $\mathbf{U}_{e,\mu}^{\mathcal{H}}\mathbf{A}_{e,\mu}^{\mathcal{H}}\mathbf{A}_{e,\mu}\mathbf{U}_{e,\mu} = \Lambda_{e,\mu}$, where $\Lambda_{e,\mu}$ is diagonal with non-increasing diagonal entries collected in the vector $\boldsymbol{\lambda}_{e,\mu} := [\lambda_{e,\mu}(0), \lambda_{e,\mu}(1), \ldots, \lambda_{e,\mu}(\tilde{L}_\mu)]^T$.

Consider now the channel vectors $\bar{\mathbf{h}}'_\mu := \mathbf{U}_{e,\mu}^{\mathcal{H}}\bar{\mathbf{h}}_\mu$, with identity correlation matrix. The vector $\bar{\mathbf{h}}'_\mu$ is clearly zero-mean, complex Gaussian, with i.i.d entries. Using $\bar{\mathbf{h}}'_\mu$, we can rewrite (17) as:

$$d^2(\mathbf{z},\mathbf{z}') = \sum_{\mu=1}^{2}(\bar{\mathbf{h}}'_\mu)^{\mathcal{H}}\mathbf{U}_{e,\mu}^{\mathcal{H}}\mathbf{A}_{e,\mu}^{\mathcal{H}}\mathbf{A}_{e,\mu}\mathbf{U}_{e,\mu}\bar{\mathbf{h}}'_\mu = \sum_{l=1}^{\tilde{L}_1}\lambda_{e,1}(l)|\bar{h}'_1(l)|^2 + \sum_{l=1}^{\tilde{L}_2}\lambda_{e,2}(l)|\bar{h}'_2(l)|^2. \tag{18}$$

Based on (18), and by averaging (14) with respect to the i.i.d. Rayleigh random variables $|\bar{h}'_1(l)|, |\bar{h}'_2(l)|$, we can upper bound the average PEP as follows:

$$P(\mathbf{s}\to\mathbf{s}') \le \prod_{l=0}^{\tilde{L}_1}\frac{1}{1+\lambda_{e,1}(l)/(4N_0)}\prod_{l=0}^{\tilde{L}_2}\frac{1}{1+\lambda_{e,2}(l)/(4N_0)}. \tag{19}$$

If $r_{e,\mu}$ is the rank of $\mathbf{A}_{e,\mu}$ (and thus the rank of $\mathbf{A}_{e,\mu}^{\mathcal{H}}\mathbf{A}_{e,\mu}$), then $\lambda_{e,\mu}(l) \neq 0$ if and only if $l \in [0, r_{e,\mu}-1]$. It thus follows from (19) that

$$P(\mathbf{s}\to\mathbf{s}') \le \left(\frac{1}{4N_0}\right)^{-(r_{e,1}+r_{e,2})}\left(\prod_{l=0}^{r_{e,1}-1}\lambda_{e,1}(l)\prod_{l=0}^{r_{e,2}-1}\lambda_{e,2}(l)\right)^{-1}. \tag{20}$$

We call $r_e := r_{e,1}+r_{e,2}$ the diversity gain $G_{d,e}$, and $\left[\prod_{l=0}^{r_{e,1}-1}\lambda_{e,1}(l)\prod_{l=0}^{r_{e,2}-1}\lambda_{e,2}(l)\right]^{1/r_e}$ the coding gain $G_{c,e}$ of the system for a given symbol error vector $\mathbf{e}$. The diversity gain $G_{d,e}$ determines the slope of the averaged (w.r.t. the random channel) PEP (between $\mathbf{s}$ and $\mathbf{s}'$) as a function of the signal to noise ratio (SNR) at high SNR ($N_0 \to 0$). Correspondingly, $G_{c,e}$ determines the shift of this PEP curve in SNR relative to a benchmark error rate curve of $[1/(4N_0)]^{-r_e}$. Without relying on PEP to design (nonlinear) ST codes for flat fading channels, we here invoke PEP bounds to prove diversity properties of our proposed single-carrier block transmissions over frequency selective channels.

Since both $G_{d,e}$ and $G_{c,e}$ depend on the choice of e (thus on s and s′), we define the diversity and coding gains for our system, respectively, as:

$$G_d := \min_{e \neq 0} G_{d,e}, \quad \text{and} \quad G_c := \min_{e \neq 0} G_{c,e}. \tag{21}$$

Based on (21), one can check both diversity and coding gains. However, in this paper, we focus only on the diversity gain. First, we observe that the matrix $\mathbf{A}_{e,\mu}^{\mathcal{H}}\mathbf{A}_{e,\mu}$ is square of size $(\tilde{L}_\mu + 1)$. Therefore, the maximum achievable diversity gain in a two transmit- and one receive-antennae system is $G_d = \sum_{\mu=1}^{2}(\tilde{L}_\mu + 1)$ for FIR channels with effective channel order $\tilde{L}_\mu$, $\mu = 1, 2$, while it becomes $2(L+1)$ in rich scattering environments. This maximum diversity can be easily achieved by e.g., a simple redundant transmission where each antenna transmits the same symbol followed by L zeros in two non-overlapping time slots. We next examine the achieved diversity levels in our following proposed schemes, which certainly have much higher rate than redundant transmissions.

B. CP-only

We term CP-only the block transmissions with no precoding: $\Theta = \mathbf{I}_K$, $J = K$, and $\mathbf{s}(i) = \mathbf{d}(i)$. The word "only" emphasizes that, unlike OFDM, no IFFT is applied at the transmitter. Let us now check the diversity order achieved by CP-only. The worst case is to select $\mathbf{d} = a\mathbf{1}_{J\times 1}$ and $\mathbf{d}' = a'\mathbf{1}_{J\times 1}$ implying $\mathbf{e} = (a - a')\mathbf{1}_{J\times 1}$, where $a, a' \in \mathcal{A}$. Verifying that for these error events, the matrix $\mathbf{D}_e = \mathrm{diag}(\mathbf{F}_J\mathbf{e})$ has only one non-zero entry, we deduce that $r_{e,1} = r_{e,2} = 1$. Therefore, the system diversity order achieved by CP-only is $G_d = 2$. This is nothing but space-diversity of order two coming from the two transmit antennas [c.f. (13)]. Note that CP-only schemes suffer from loss of multipath diversity.

To benefit also from the embedded multipath-induced diversity, we have to modify our transmissions.

C. Linearly Precoded CP-only

To increase our ST system's diversity order, transmitter 4 may utilize linear precoding developed originally for single-antenna transmissions. One can view CP-only as a special case of the linearly precoded CP-only system (denoted henceforth as LP-CP-only) with identity precoder. With s(i) = Θd(i) and carefully designed $\Theta \neq I_K$, we next show that the maximum diversity is achieved. We will discuss two cases: the first one introduces no redundancy because it uses J = K, while the second one is redundant and adopts J = K + L. For non-redundant precoding with J = K, it has been established that for *any* signal constellation adhering to a finite alphabet, there always exists a K x K unitary constellation rotating (CR) matrix ensuring Θ_{CR} that each entry of Θ_{CR} (d - d ′) is non-zero *for any* pair of (d, d ′). We thus propose to

construct $\Theta = \mathbf{F}_K^{\mathcal{H}}\Theta_{cr}$ such that $\mathbf{F}_K\Theta = \Theta_{cr}$. With this construction, $\mathbf{D}_e = \mathrm{diag}(\Theta_{cr}\mathbf{e})$ is guaranteed to have non-zero entries on its diagonal, and thus it has full rank. Consequently, the matrix $\mathbf{D}_e\mathbf{V}$ has full column rank $L+1$, and $\mathbf{A}_{e,\mu} = \mathbf{D}_e\mathbf{V}\mathbf{U}_{h,\mu}\Lambda_{h,\mu}^{\frac{1}{2}}$ has full column rank $r_{e,\mu} = \bar{L}_\mu + 1$. Hence, the maximum achievable diversity order is indeed achieved.

We emphasize here that the non-redundant precoder Θ_{cr} is constellation dependent. For commonly used BPSK, QPSK, and all QAMs constellations, and for the block size K equal to a power of 2: $K = 2^m$, one class of Θ_{cr} precoders with large coding gains is found to be :

$$\Theta_{cr} = \mathbf{F}_K\Delta(\alpha), \quad \text{and thus,} \quad \Theta = \Delta(\alpha), \tag{22}$$

where $\Delta(\alpha) := \mathrm{diag}(1, \alpha, \ldots, \alpha^{K-1})$ with $\alpha \in \{e^{j\frac{\pi}{2K}(1+4n)}\}_{n=0}^{K-1}$. For block size $K \neq 2^m$, one can construct Θ_{cr} by truncating a larger unitary matrix constructed as in (22) . The price paid for our increased diversity gain is that LP-CP-only does not offer constant modulus transmissions, in general. However, by designing K to be a power of 2, and by choosing Θ as in (22), the transmitted signals $\mathbf{s}(i) = \Delta(\alpha)\mathbf{d}(i)$ are constant modulus if $\mathbf{d}(i)$ are PSK signals. Therefore, by selecting K to be a power of 2, we can increase the diversity gain without reducing power efficiency.

Alternatively, we can adopt a redundant $J \times K$ precoder Θ with $J = K+L$. Our criterion for selecting such tall precoding matrices Θ is to guarantee that $\mathbf{F}_J\Theta$ satisfies the following property: *any* K rows of $\mathbf{F}_J\Theta$ are linearly independent. One class of $\mathbf{F}_J\Theta$ satisfying this property includes Vandermonde matrices Θ_{van} with distinct generators $[\rho_1, \ldots, \rho_J]$, defined as :

$$\Theta_{van} = \frac{1}{\sqrt{J}}\begin{bmatrix} 1 & \rho_1^{-1} & \cdots & \rho_1^{-(K-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \rho_J^{-1} & \cdots & \rho_J^{-(K-1)} \end{bmatrix}, \quad \text{and thus,} \quad \Theta = \mathbf{F}_J^{\mathcal{H}}\Theta_{van}. \tag{23}$$

With $\mathbf{F}_J\Theta = \Theta_{van}$, we have that $\Theta_{van}\mathbf{e}$ has at least $(L+1)$ nonzero entries for any $\mathbf{e}$ regardless of the underlying signal constellation. Indeed, if $\Theta_{van}\mathbf{e}$ has only L nonzero entries for some $\mathbf{e}$, then it has K zero entries. Picking the corresponding K rows of Θ_{van} to form the truncated matrix $\bar{\Theta}_{van}$, we have $\bar{\Theta}_{van}\mathbf{e} = \mathbf{0}$, which shows that these K rows are linearly dependent, thus violating the design of the precoder Θ_{van}. With $\mathbf{D}_e = \mathrm{diag}(\Theta_{van}\mathbf{e})$ having at least $(L+1)$ nonzero entries, the matrix $\mathbf{D}_e\mathbf{V}$ has full rank because *any* $L+1$ rows of $\mathbf{V}$ are linearly independent. Thus, the maximum diversity gain is achieved with redundant precoding *irrespective* of the underlying constellation.

When $J \in [K, K+L]$, constellation irrespective precoders are impossible because $\Theta\mathbf{e}$ can not have $L+1$ nonzero entries for any $\mathbf{e}$ that is unconstrained. Therefore, constellation independent precoders are not possible for J < K + L. However, with some redundancy J > K, the design of constellation-dependent precoders may become easier.

D. Affine Precoded CP-only

Another interesting class of linear precoders implements an affine transformation: $\mathbf{s}(i) = \Theta\mathbf{d}(i) + \Theta'\mathbf{b}(i)$, where $\mathbf{b}(i)$ is a known symbol vector. In this paper, we are only interested in the special form of:

$$\mathbf{s}(i) = \mathbf{T}_1\mathbf{d}(i) + \mathbf{T}_2\mathbf{b}(i) = \begin{bmatrix}\mathbf{d}(i)\\ \mathbf{b}(i)\end{bmatrix}, \tag{24}$$

where the precoder $\Theta = \mathbf{T}_1$ is the first K columns of $\mathbf{I}_J$, the precoder $\Theta' = \mathbf{T}_2$ is the last L columns of $\mathbf{I}_J$, and the known symbol vector $\mathbf{b}$ has size $L \times 1$ with entries drawn from the same alphabet $\mathcal{A}$. We henceforth term the transmission format in (24) as AP-CP-only. Notice that in this scheme, $J = K + L$ and $P = J + L$.

Although here we place $\mathbf{b}(i)$ at the bottom of $\mathbf{s}(i)$ for convenience, we could also place $\mathbf{b}(i)$ at arbitrary positions within $\mathbf{s}(i)$. As long as L consecutive symbols are known in $\mathbf{s}(i)$, all decoding schemes detailed in Section II are applicable.

Recall that the error matrix $\mathbf{D}_e = \mathrm{diag}(\mathbf{F}_J\mathbf{T}_1\mathbf{e})$ does not contain known symbols. Since $\mathbf{F}_J\mathbf{T}_1$ is a Vandermonde matrix of the form (23), the maximum diversity gain is achieved, as discussed in Section I-C for redundant LP-CP-only.

In the CP-based schemes depicted in Fig. 2, the CP portion of the transmitted sequence is generally unknown, because it is replicated from the unknown data blocks. However, with AP-CP-only in (24), and with the specific choice of $\mathbf{P} = \mathbf{P}_J^{(K)}$, we have $\mathbf{P}_J^{(K)}\mathbf{s}(i) = [[\mathbf{P}_K^{(0)}\mathbf{d}(i)]^T, \ [\mathbf{P}_L^{(0)}\mathbf{b}(i)]^T]^T$, which implies that *both* the data block and the known symbol block are time reversed, but keep their original positions. The last L entries of $\mathbf{P}_J^{(K)}\mathbf{s}(i)$ are again known, and are then replicated as cyclic prefixes. For this *special case*, we depict the transmitted sequences in Fig. 3. In this format, the data block $\mathbf{d}(i)$ is surrounded by two known blocks, that correspond to the pre-amble and post-amble. Our general design based on the CP structure includes this known pre- and post- ambles as a special case.

Notice that the pre-amble and post-amble have not been properly designed in some conventional systems. The consequence is that "edge effects" appear for transmissions with finite block length, and an approximation on the order of O (L/J) has to be made in order to apply Viterbi's decoding algorithm. This approximation amounts to nothing but the fact that a linear convolution can be approximated by a circular convolution when the block size is much larger than the channel order. By simply enforcing a CP structure to obtain circulant convolutions, Viterbi's algorithm can be applied to our proposed AP-CP-only with *no approximation* whatsoever, regardless of the block length and the channel order, as will be clear soon.

E. ZP-only

Suppose now that in AP-CP-only, we let $\mathbf{b}(i) = \mathbf{0}$ instead of having known symb ls drawn from the constellation alphabet, and we fix $\mathbf{P} = \mathbf{P}_J^{(K)}$. Now, the adjacent data blocks are guarded by two zero blocks, each having length L, as depicted in Fig. 3. Since the channel has only order L, presence of $2L$ zeros in the middle of two adjacent data blocks is not necessary. Keeping only a single block of L zeros corresponds to removing the CP-insertion operation at the transmitter. On the other hand, one could view that the zero block in the previous block serves as the CP for the current block, and thus all derivations done for CP-based transmission are still valid. The resulting transmission format is shown in Fig. 4, which achieves higher bandwidth efficiency than AP-CP-only. We term this scheme as ZP-only, where $J = K + L$ and $P = J$.

By mathematically viewing ZP-only as a special case of AP-CP-only with $\mathbf{b}(i) = \mathbf{0}$, it is clear that the maximum diversity is achieved. In addition to the rate improvement, ZP-only also saves the transmitted power occupied by CP and known symbols.

For convenience, we list all aforementioned schemes in Table I, assuming a rich scattering environment. Power loss induced by the cyclic prefix and the known symbols, is also considered. It certainly becomes negligible when $K \gg L$.

F. Links with multicarrier transmissions

In this section, we link single carrier with digital multicarrier (OFDM based) schemes. We first examine the transmitted blocks on two consecutive time intervals. For LP-CP-only, the transmitted space-time matrix is:

$$\begin{bmatrix} \mathbf{u}_1(2i) & \mathbf{u}_1(2i+1) \\ \mathbf{u}_2(2i) & \mathbf{u}_2(2i+1) \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{cp}\Theta\mathbf{d}(2i) & -\mathbf{T}_{cp}\mathbf{P}\Theta^*\mathbf{d}^*(2i+1) \\ \mathbf{T}_{cp}\Theta\mathbf{d}(2i+1) & \mathbf{T}_{cp}\mathbf{P}\Theta^*\mathbf{d}^*(2i) \end{bmatrix} \begin{array}{l} \rightarrow \text{time} \\ \downarrow \text{space} \end{array}, \tag{25}$$

If let $\mathbf{P} = \mathbf{P}_J^{(1)}$ and $\Theta = \mathbf{F}_J^{\mathcal{H}}\Psi$, we obtain for a general matrix Ψ:

$$\begin{bmatrix} \mathbf{u}_1(2i) & \mathbf{u}_1(2i+1) \\ \mathbf{u}_2(2i) & \mathbf{u}_2(2i+1) \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{cp}\mathbf{F}_J^{\mathcal{H}}\Psi\mathbf{d}(2i) & -\mathbf{T}_{cp}\mathbf{F}_J^{\mathcal{H}}\Psi^*\mathbf{d}^*(2i+1) \\ \mathbf{T}_{cp}\mathbf{F}_J^{\mathcal{H}}\Psi\mathbf{d}(2i+1) & \mathbf{T}_{cp}\mathbf{F}_J^{\mathcal{H}}\Psi^*\mathbf{d}^*(2i) \end{bmatrix} \begin{array}{l} \rightarrow \text{time} \\ \downarrow \text{space} \end{array}. \tag{26}$$

TABLE I

SUMMARY OF SINGLE CARRIER SCHEMES IN RICH-SCATTERING ENVIRONMENTS

	Rate R	Diversity G_d	Power Loss (dB)	Features
CP-only	$\frac{K}{K+L}\log_2\lvert\mathcal{A}\rvert$	2	$10\log_{10}\frac{K+L}{K}$	constant modulus (C-M)*
non-redundant LP-CP-only	$\frac{K}{K+L}\log_2\lvert\mathcal{A}\rvert$	$2(L+1)$	$10\log_{10}\frac{K+L}{K}$	constellation-specific precoder constant modulus
redundant LP-CP-only	$\frac{K}{K+2L}\log_2\lvert\mathcal{A}\rvert$	$2(L+1)$	$10\log_{10}\frac{K+L}{K}$	constellation-independent Not C-M in general
AP-CP-only	$\frac{K}{K+2L}\log_2\lvert\mathcal{A}\rvert$	$2(L+1)$	$10\log_{10}\frac{K+2L}{K}$	constellation-independent constant modulus
ZP-only	$\frac{K}{K+L}\log_2\lvert\mathcal{A}\rvert$	$2(L+1)$	0	constellation-independent C-M except zero guards

* - only if information symbols have constant-modulus, e.g., drawn from PSK constellations.

If $\Psi = \mathbf{I}_K$, then (26) corresponds to the space-time block coded OFDM proposed in Y.Li,J.C.Chung,andN.R.Sollenberger,"Transmitter diversity for OFDM systems and its impact on high-rate data wireless networks," IEEE Journal on Selected Areas in Communications, vol. 17, no.7, pp.1233–1243, July1999. Designing $\Psi \neq \mathbf{I}_K$ introduces linear precoding across OFDM subcarriers, as proposed in other conventional techniques. Therefore, LP- CP-only includes linear precoded space-time OFDM as a special case by selecting the precoder $\Phi$and the permutation $\mathbf{P}$ appropriately. Although linear precoding has been proposed for space time OFDM systems, the diversity analysis has not been provided. The link we introduce here reveals that the maximum diversity gain is also achieved by linearly precoded ST-OFDM with the Vandermonde precoders.

Interestingly, linearly precoded OFDM can even be converted to zero padded transmissions. Indeed, choosing Ψ to be the first K columns of $\mathbf{F}_J$, we obtain the transmitted block as: $\mathbf{u}(i) = \mathbf{T}_{cp}\mathbf{F}_J^{\mathcal{H}}\Psi\mathbf{d}(i) = [\mathbf{0}_{L\times 1}^T, \mathbf{d}^T(i), \mathbf{0}_{L\times 1}^T]^T$, which inserts zeros both at the top and at the bottom of each data block.

G. Capacity Result

We now analyze the capacity of the space time block coding format of (1). The equivalent channel input-output relationship, after receiver processing, is described by (13) as: $\mathbf{z} = \bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{s} + \boldsymbol{\eta}$, where we drop the block index for brevity. Let $\mathcal{I}(\mathbf{z};\mathbf{s})$ denote the mutual information between $\mathbf{z}$ and $\mathbf{s}$, and recall that $\mathcal{I}(\mathbf{z};\mathbf{s})$ is maximized when $\mathbf{s}$ is Gaussian distribute. Due to the lack of channel knowledge at the transmitter, the transmission power is equally distributed among symbols, with $\mathbf{R}_s = E\{\mathbf{s}\mathbf{s}^{\mathcal{H}}\} = \sigma_s^2\mathbf{I}_J$. Taking into account the CP of length L, the channel capacity, for a fixed channel realization, is thus:

$$\begin{aligned} C_J &= \frac{1}{J+L}\max\mathcal{I}(\mathbf{z};\mathbf{s}) = \frac{1}{J+L}\log_2\det\left(\mathbf{I}_J + \frac{\sigma_s^2}{N_0}\bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{F}_J^{\mathcal{H}}\bar{\mathcal{D}}_{12}\right) \\ &= \frac{1}{J+L}\sum_{n=0}^{J-1}\log_2\left(1 + \frac{\sigma_s^2}{N_0}(|H_1(e^{j\frac{2\pi}{J}n})|^2 + |H_2(e^{j\frac{2\pi}{J}n})|^2)\right). \end{aligned} \tag{27}$$

Define $E_s = 2\sigma_s^2$ as the total transmitted power from two antennas per channel use. As the block size J increases, we obtain

$$C_{J\to\infty} = \int_0^1 \log_2\left(1 + \frac{E_s}{2N_0}(|H_1(e^{j2\pi f})|^2 + |H_2(e^{j2\pi f})|^2)\right) df. \tag{28}$$

The capacity for frequency selective channels with multiple transmit and receive antennas has been described with conventional techniques. The result in (28) coincides with that of some of these techniques when we have two transmit antennas and one receive antenna. Therefore, our proposed transmission format in (1) does not incur capacity loss in this special case. This is consistent with techniques where the Alamouti coding is shown to achieve capacity for frequency-flat fading channels with such an antenna configuration. To achieve capacity for systems with two transmit antennas and a single receive antenna, it thus suffices to deploy suitable one-dimensional channel codes, or scalar codes.

II. EQUALIZATION AND DECODING

Let $\bar{\mathbf{z}}(i) := \mathbf{z}(i)$ for CP-only, LP-CP-only, ZP-only, and $\bar{\mathbf{z}}(i) := \mathbf{z}(i) - \bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{T}_2\mathbf{b}(i)$ for AP-CP-only. With this convention, we can unify the equivalent system output after the linear receiver processing as:

$$\bar{\mathbf{z}}(i) = \mathbf{F}_J\Theta\mathbf{d}(i) + \boldsymbol{\eta}(i) = \mathbf{A}\mathbf{d}(i) + \boldsymbol{\eta}(i), \tag{29}$$

where $\mathbf{A} := \mathbf{F}_J\Theta$, the noise $\boldsymbol{\eta}(i)$ is white with covariance $\sigma_w^2\mathbf{I}_J$, and the corresponding Θ is defined as in Section I.

Brute-force ML decoding applied to (29) requires $|\mathcal{A}|^K$ enumerations, which becomes certainly prohibitive as the constellation size $|\mathcal{A}|$ and/or the block length K increases. A relatively faster near-ML search is possible with the sphere decoding (SD) algorithm, which only searches for vectors that are within a sphere centered at the received symbols. The theoretical complexity of SD is polynomial in K, which is lower than exponential, but still too high for $K > 16$. Only when the block size K is small, the SD equalizer can be adopted to achieve near-ML performance at a manageable complexity. The unique feature of SD is that the complexity does not depend on the constellation size. Thus, SD is suitable for systems with small block size K, but with large signal constellations.

We now turn our attention to low-complexity equalizers by trading off performance with complexity. Linear zero forcing (ZF) and minimum mean square error (MMSE) block equalizers certainly offer low complexity alternatives. The block MMSE equalizer is:

$$\mathbf{\Gamma}_{mmse} = (\mathbf{A}^{\mathcal{H}}\mathbf{A} + \sigma_w^2/\sigma_s^2\mathbf{I}_K)^{-1}\mathbf{A}^{\mathcal{H}}, \tag{30}$$

where we have assumed that the symbol vectors are white with covariance matrix $\mathbf{R}_s = E\{\mathbf{s}(i)\mathbf{s}^{\mathcal{H}}(i)\} = \sigma_s^2\mathbf{I}_K$. The MMSE equalizer reduces to the ZF equalizer by setting $\sigma_w^2 = 0$ in (30).

For non-redundant LP-CP-only with $\Theta = \mathbf{\Delta}(\alpha)$, we further simplify (30) to

$$\mathbf{\Gamma}_{mmse} = \mathbf{\Delta}(\alpha^*)\mathbf{F}_K^{\mathcal{H}}[\bar{\mathcal{D}}_{12}^2 + \sigma_w^2/\sigma_s^2\mathbf{I}_K]^{-1}\bar{\mathcal{D}}_{12}, \tag{31}$$

A. ML decoding for AP-CP-only and ZP-only

For AP-CP-only and ZP-only, we have

$$\mathbf{z} = \bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{s} + \boldsymbol{\eta}, \tag{32}$$

where we drop the block index i for simplicity. Distinct from other systems, AP-CP-only and ZP-only assure that $\mathbf{s}$ has the last L entries known, and the first K entries drawn from the finite alphabet $\mathcal{A}$.

In the presence of white noise, ML decoding can be expressed as:

$$\hat{\mathbf{s}}_{ML} = \arg\max_{\mathbf{s}} \ln P(\mathbf{z}|\mathbf{s}) = \arg\max_{\mathbf{s}} \{-\|\mathbf{z} - \bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{s}\|^2/N_0\}. \tag{33}$$

We next simplify (33), starting with

$$\begin{aligned} -\|\mathbf{z} - \bar{\mathcal{D}}_{12}\mathbf{F}_J\mathbf{s}\|^2 &= 2\mathrm{Re}\{\mathbf{s}^{\mathcal{H}}\mathbf{F}_J^{\mathcal{H}}\bar{\mathcal{D}}_{12}\mathbf{z}\} - \mathbf{s}\mathbf{F}_J^{\mathcal{H}}\bar{\mathcal{D}}_{12}^2\mathbf{F}_J\mathbf{s} - \mathbf{z}^{\mathcal{H}}\mathbf{z} \\ &= 2\mathrm{Re}\{\mathbf{s}^{\mathcal{H}}\mathbf{r}\} - \sum_{\mu=1}^{2}\|\bar{\mathbf{H}}_\mu\mathbf{s}\|^2 - \mathbf{z}^{\mathcal{H}}\mathbf{z}, \end{aligned} \tag{34}$$

where $\mathbf{r} := \mathbf{F}_J^{\mathcal{H}}\bar{\mathcal{D}}_{12}\mathbf{z}$. We let $r_n := [\mathbf{r}]_n$, and $s_n := [\mathbf{s}]_n$. Recognizing that $\bar{\mathbf{H}}_\mu\mathbf{s}$ expresses nothing but a circular convolution between the channel $\mathbf{h}$ and $\mathbf{s}$, we have $[\bar{\mathbf{H}}_\mu\mathbf{s}]_n = \sum_{l=0}^{L} h_\mu(l)s_{(n-l) \bmod J}$. Hence, we obtain:

$$\hat{\mathbf{s}}_{ML} = \arg\max_{\mathbf{s}} \sum_{n=0}^{J-1}\left\{\frac{1}{N_0}\left[2\,\mathrm{Re}\{s_n^* r_n\} - \sum_{\mu=1}^{2}|\textstyle\sum_{l=0}^{L} h_\mu(l)s_{(n-l)\bmod J}|^2\right]\right\}. \tag{35}$$

For each $n = 0, 1, \ldots, J$, let us define a sequence of state vectors as: $\zeta_n = [s_{(n-1)\bmod J}, \ldots, s_{(n-L)\bmod J}]^T$, out of which the first and the last states are known: $\zeta_0 = \zeta_J = [s_{J-1}, \ldots, s_{J-L}]^T$. The symbol sequence $s_0, \ldots, s_{J-1}$ determines an unique path evolving from the known initial state ζ_0 to the known final state ζ_J. Thus, Viterbi's algorithm is applicable. Specifically, we have:

$$\hat{\mathbf{s}}_{ML} = \arg\max_{\mathbf{s}} \sum_{n=0}^{J-1} f(\zeta_n, \zeta_{n+1}), \tag{36}$$

where $f(\zeta_n, \zeta_{n+1})$ is the branch metric, that is readily obtainable from (35). The explicit recursion formula for Viterbi's Algorithm is well known.

We now simplify the branch metric further. We first have $\sum_{\mu=1}^{2}\|\bar{\mathbf{H}}_\mu\mathbf{s}\|^2 = \mathbf{s}^{\mathcal{H}}\sum_{\mu=1}^{2}(\bar{\mathbf{H}}_\mu^{\mathcal{H}}\bar{\mathbf{H}}_\mu)\mathbf{s}$. The matrix $\bar{\mathbf{H}} := \sum_{\mu=1}^{2}(\bar{\mathbf{H}}_\mu^{\mathcal{H}}\bar{\mathbf{H}}_\mu)$ has (p,q)th entry:

$$[\bar{\mathbf{H}}]_{p,q} = \sum_{\mu=1}^{2}\sum_{n=0}^{J-1} h_\mu^*((k-p)\bmod J)h_\mu((k-q)\bmod J). \tag{37}$$

Let us now select $J > 2L$, and define

$$\beta_n = \sum_{\mu=1}^{2}\sum_{l=0}^{L} h_\mu^*(l)h_\mu(n+l), \text{ for } n = 0, 1, \ldots, L. \tag{38}$$

It can be easily verified that the first column of $\tilde{\mathbf{H}}$ is $[\beta_0, \beta_1, \ldots, \beta_L, 0, \ldots, 0, \beta_L^*, \ldots, \beta_1^*]^T$. Let $\check{\mathbf{H}}$ denote the circulant matrix with first column $[(1/2)\beta_0, \beta_1, \ldots, \beta_L, 0, \ldots, 0]^T$. Because $\tilde{\mathbf{H}}$ is circulant and Hermitian, $\tilde{\mathbf{H}}$ can be decomposed into: $\tilde{\mathbf{H}} = \check{\mathbf{H}} + \check{\mathbf{H}}^{\mathcal{H}}$. We thus obtain $\mathbf{s}^{\mathcal{H}}\tilde{\mathbf{H}}\mathbf{s} = 2\mathrm{Re}\{\mathbf{s}^{\mathcal{H}}\check{\mathbf{H}}\mathbf{s}\}$. Recognizing $[\check{\mathbf{H}}\mathbf{s}]_n = (1/2)\beta_0 s_n + \sum_{l=1}^{L}\beta_l s_{(n-l) \bmod J}$, and combining with (35), we obtain a simplified metric as:

$$f(\zeta_n, \zeta_{n+1}) = \frac{2}{N_0}\mathrm{Re}\left\{ s_n^*\left[r_n - \frac{1}{2}\beta_0 s_n - \sum_{l=1}^{L}\beta_l s_{(n-l) \bmod J}\right]\right\}. \tag{39}$$

The branch metric in (39) has a format analogous to the one proposed by Ungerboeck for maximum- likelihood sequence estimation (MLSE) receivers with *single antenna serial* transmissions. For multi-antenna block coded transmissions, a similar metric has been suggested in conventional systems. The systems, however, can suffer from "edge effects" for transmissions with finite block length, resulting an approximation on the order of O (L/J), while our derivation here is exact. Our CP based design assures a circular convolution, while the linear convolution in some conventional systems approximates well a circulant convolution only when J >> L. Note also that we allow for an arbitrary permutation matrix **P**, which includes the time-reversal in as a special case. Furthermore, a known symbol vector b can be placed in an arbitrary position within the vector s for AP-CP-only. If the known symbols occupy positions B – L, … , B - 1, we just need to redefine the states as

$$\zeta_n = \left[s_{(n+B-1) \bmod J}, \ldots, s_{(n+B-L) \bmod J}\right]^T.$$

Notice that for channels with order L, the complexity of Viterbi's algorithm is O ($|A|^L$) per symbol; thus, ML decoding with our exact application of Viterbi's algorithm should be particularly attractive for transmissions with small constellation size, over relatively short channels.

B. Turbo equalization for coded AP-CP-only and ZP-only

So far, we have only considered uncoded systems, and established that full diversity is achieved. To further improve system performance by enhancing also coding gains, conventional channel coding can be applied to our systems. For example, outer convolutional codes can be used in AP-CP-only and ZP-only, as

depicted in FIG. 5. Other codes such as TCM and turbo codes are applicable as well.

In the presence of frequency selective channels, iterative (turbo) equalization is known to enhance system performance, at least for single antenna transmissions. We here derive turbo equalizers for our coded AP-CP-only and ZP-only multi-antenna systems.

To enable turbo equalization, one needs to find the *a posteriori* probability on the transmitted symbols s_n based on the received vector $\mathbf{z}$. Suppose each constellation point s_n is determined by $Q = \log_2 |\mathcal{A}|$ bits $\{c_{n,0}, \ldots, c_{n,Q-1}\}$. Let us consider the log likelihood ratio (LLR):

$$\mathcal{L}_{n,q} = \ln \frac{P(c_{n,q} = +1|\mathbf{z})}{P(c_{n,q} = -1|\mathbf{z})}, \qquad \forall n \in [0, J-1],\ q \in [0, Q-1]. \tag{40}$$

The log-likelihood ratio in (40) can be obtained by running two generalized Viterbi recursions: one in the forward direction and one in the backward direction period. Our branch metric is modified as follows:

$$g(\zeta_n, \zeta_{n+1}) = f(\zeta_n, \zeta_{n+1}) + \ln P(\zeta_{n+1}|\zeta_n). \tag{41}$$

This modification is needed to take into account the *a priori* probability $P(\zeta_{n+1}|\zeta_n)$, determined by the extrinsic information from the convolutional channel decoders during the turbo iteration. When the transition from ζ_n to ζ_{n+1} is caused by the input symbol s_n, we have $\ln P(\zeta_{n+1}|\zeta_n) = \ln P(s_n)$. We assume that the bit interleaver in Fig. 5 renders the symbols s_n independent and equal likely, such that $\ln P(s_n) = \sum_{q=0}^{Q-1} \ln P(c_{n,q})$, which in turn can be determined by the LLRs for bits $\{c_{n,q}\}_{q=0}^{Q-1}$.

Finally, we remark that one could also adopt the known turbo decoding algorithm that is based on MMSE equalizers. This iterative receiver is applicable not only to AP-CP-only and ZP-only, but also to CP-only and LP-CP-only systems.

C. Receiver Complexity

Omitting the complexity of permutation and diagonal matrix multiplication, the linear processing to reach (13) only requires one size-J FFT per block, which amounts to $\mathcal{O}(\log_2 J)$ per information symbol.

Channel equalization is then performed based on (13) for each block. We notice that the complexity is the same as the equalization complexity for single antenna block transmissions over FIR channels [43]. We refer the readers to [43] for detailed complexity comparisons of the different equalization options. For coded AP-CP-only and ZP-only, the complexity of turbo equalization is again the same as that of single antenna transmissions [13].

In summary, the overall receiver complexity for the two transmit antenna case is comparable to that of single antenna transmissions, with only one additional FFT per data block. This nice property originates from the orthogonal space-time block code design, that enables linear ML processing to collect antenna diversity. Depending desirable/affordable diversity-complexity tradeoffs, the designer is then provided with the flexibility to collect extra multipath-diversity gains.

III. EXTENSION TO MULTIPLE ANTENNAS

In Section I, we focused on $N_t = 2$ transmit- and $N_r = 1$ receive- antennae. In this section, we will extend our system design to the general case with $N_t > 2$ and/or $N_r > 1$ antennas. For each $\mu = 1, \ldots, N_t$ and $\nu = 1, \ldots, N_r$, we denote the channel between the μth transmit- and the νth receive-antennae as $\mathbf{h}_{\mu\nu} := [h_{\mu\nu}(0), \ldots, h_{\mu\nu}(L)]^T$, and as before we model it as a zero-mean, complex Gaussian vector with covariance matrix $\mathbf{R}_{h,\mu\nu}$. Correspondingly, we define the effective channel order $\bar{L}_{\mu\nu} :=$ rank$\{\mathbf{R}_{h,\mu\nu}\} - 1$, which for a sufficiently rich scattering environment becomes $\bar{L}_{\mu\nu} = L$.

Transmit diversity with $N_t > 2$ has been addressed in for OFDM based multicarrier transmissions over FIR channels by applying the orthogonal ST block codes of on each OFDM subcarrier. Here, we extend the orthogonal designs to single carrier block transmissions over frequency selective channels.

We will review briefly generalized orthogonal designs to introduce notation, starting with the basic definitions given in the context of frequency-flat channels :

Definition 1: Define $\mathbf{x} := [x_1, \ldots, x_{N_s}]^T$, and let $\mathcal{G}_r(\mathbf{x})$ be an $N_d \times N_t$ matrix with entries $0, \pm x_1, \ldots, \pm x_{N_s}$. If $\mathcal{G}_r^T(\mathbf{x})\mathcal{G}_r(\mathbf{x}) = \alpha(x_1^2 + \cdots + x_{N_s}^2)\mathbf{I}_{N_t}$ with α positive, then $\mathcal{G}_r(\mathbf{x})$ is termed a generalized real orthogonal design (GROD) in variables $x_1, \ldots, x_{N_s}$ of size $N_d \times N_t$ and rate $R = N_s/N_d$.

Definition 2: Define $\mathbf{x} := [x_1, \ldots, x_{N_s}]^T$, and let $\mathcal{G}_c(\mathbf{x})$ be an $N_d \times N_t$ matrix with entries $0, \pm x_1, \pm x_1^*, \ldots, \pm x_{N_s}, \pm x_{N_s}^*$. If $\mathcal{G}_c^{\mathcal{H}}(\mathbf{x})\mathcal{G}_c(\mathbf{x}) = \alpha(|x_1|^2 + \cdots + |x_{N_s}|^2)\mathbf{I}_{N_t}$ with α positive, then $\mathcal{G}_c(\mathbf{x})$ is termed a generalized real orthogonal design (GCOD) in variables $x_1, \ldots, x_{N_s}$ of size $N_d \times N_t$ and rate $R = N_s/N_d$.

Explicit construction of $\mathcal{G}_r(\mathbf{x})$ with $R = 1$ was discussed in [34], where it was also proved that the highest rate for $\mathcal{G}_c(\mathbf{x})$ is 1/2 when $N_t > 4$. When $N_t = 3, 4$, there exist some sporadic codes with rate $R = 3/4$. Although the orthogonal designs with $R = 3/4$ for $N_t = 3, 4$ have been incorporated for multicarrier transmissions, we will not consider them in our single carrier block transmissions here; we will only consider the $R = 1/2$ GCOD designs primarily because GCOD $\mathcal{G}_c(\mathbf{x})$ of $R = 1/2$ can be constructed using the following steps ($N_s = 4$ for $N_t = 3, 4$, while $N_s = 8$ for $N_t = 5, 6, 7, 8$ [34]):

s1) construct GROD $\mathcal{G}_r(\mathbf{x})$ of size $N_s \times N_t$ with $R = 1$;

s2) replace the symbols $x_1, \ldots, x_{N_s}$ in $\mathcal{G}_r(\mathbf{x})$ by their conjugates $x_1^*, \ldots, x_{N_s}^*$ to arrive at $\mathcal{G}_r(\mathbf{x}^*)$;

s3) form $\mathcal{G}_c(\mathbf{x}) = [\mathcal{G}_r^T(\mathbf{x}), \mathcal{G}_r^T(\mathbf{x}^*)]^T$.

As will be clear soon, we are explicitly taking into account the fact that all symbols from the upper-part of $\mathcal{G}_c(\mathbf{x})$ are un-conjugated, while all symbols from the lower-part are conjugated. The rate loss can be as high as 50%, when $N_t > 2$.

With $N_t > 2$, the space-time mapper takes N_s consecutive blocks to output the following $N_t J \times N_d$ space time coded matrix ($N_d = 2N_s$)

$$\bar{\mathbf{S}}(i) = \mathcal{E}\left\{\mathbf{s}(iN_s), \ldots, \mathbf{s}(iN_s + N_s - 1)\right\} = \begin{bmatrix} \bar{\mathbf{s}}_1(iN_d) & \cdots & \bar{\mathbf{s}}_1(iN_d + N_d - 1) \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{s}}_{N_t}(iN_d) & \cdots & \bar{\mathbf{s}}_{N_t}(iN_d + N_d - 1) \end{bmatrix} \begin{matrix} \rightarrow \text{time} \\ \\ \downarrow \text{space} \end{matrix} . \quad (42)$$

The design steps are summarized as follows:

d1) construct $\mathcal{G}_c$ of size $2N_s \times N_t$ in the variables $x_1, \ldots, x_{N_s}$, as in s1)-s3);

d2) Replace $x_1, \ldots, x_{N_s}$ in $\mathcal{G}_c^T$ by $s(iN_s), \ldots, s(iN_s + N_s - 1)$;

d3) Replace $x_1^*, \ldots, x_{N_s}^*$ in $\mathcal{G}_c^T$ by $\mathbf{P}s^*(iN_s), \ldots, \mathbf{P}s^*(iN_s + N_s - 1)$, where $\mathbf{P}$ is taken properly for different schemes as explained in Section I.

At each block transmission slot i, $\mathbf{s}_\mu(i)$ is forwarded to the μth antenna, and transmitted through the FIR channel after CP insertion. Each receive antenna processes blocks independently as follows: The receiver removes the CP, and collects $N_d = 2N_s$ blocks $\mathbf{x}(iN_d), \ldots, \mathbf{x}(iN_d + N_d - 1)$. Then FFT is performed on the first N_s blocks $\mathbf{x}(iN_d), \ldots, \mathbf{x}(iN_d + N_s - 1)$, while permutation and conjugation is applied to the last N_s blocks: $\mathbf{P}\mathbf{x}^*(iN_d + N_s), \ldots, \mathbf{P}\mathbf{x}^*(iN_d + N_d - 1)$, followed by FFT processing. Coherently combining the FFT outputs as we did for the two antennae case to derive (13), we obtain on each antenna the equivalent output after the optimal linear processing:

$$\mathbf{z}_\nu(i) = \bar{\mathcal{D}}_\nu \mathbf{F}_J \mathbf{s}(i) + \boldsymbol{\eta}_\nu(i), \tag{43}$$

where $\bar{\mathcal{D}}_\nu := [\sum_{\mu=1}^{N_t} \mathcal{D}_{\mu,\nu}^* \mathcal{D}_{\mu,\nu}]^{1/2}$ and $\mathcal{D}_{\mu,\nu} := \mathrm{diag}(\tilde{\mathbf{h}}_{\mu\nu}) = \mathrm{diag}(\mathbf{V}\mathbf{h}_{\mu\nu})$.

We next stack the $\mathbf{z}_\nu(i)$ blocks to form $\bar{\mathbf{z}}(i) = [\mathbf{z}_1^T(i), \ldots, \mathbf{z}_{N_r}^T(i)]^T$ (likewise for $\bar{\boldsymbol{\eta}}(i)$), and define $\mathbf{B} := [\bar{\mathcal{D}}_1, \ldots, \bar{\mathcal{D}}_{N_r}]^T$, to obtain: $\bar{\mathbf{z}}(i) = \mathbf{B}\mathbf{F}_J\mathbf{s}(i) + \bar{\boldsymbol{\eta}}(i)$. Defining $\bar{\mathbf{B}} := [\sum_{\mu=1}^{N_t} \sum_{\nu=1}^{N_r} \mathcal{D}_{\mu,\nu}^* \mathcal{D}_{\mu,\nu}]^{1/2}$, we have $\mathbf{B}^{\mathcal{H}}\mathbf{B} = \bar{\mathbf{B}}^2$. Therefore, we can construct a matrix $\mathbf{U}_b := \mathbf{B}\bar{\mathbf{B}}^{-1}$, which has orthonormal columns $\mathbf{U}_b^{\mathcal{H}}\mathbf{U}_b = \mathbf{I}_J$, and satisfies $\mathbf{U}_b^{\mathcal{H}}\mathbf{B} = \bar{\mathbf{B}}$. As $\mathbf{U}_b$ and $\mathbf{B}$ share range spaces, multiplying $\mathbf{U}_b^{\mathcal{H}}$ by $\bar{\mathbf{z}}(i)$ incurs no loss of optimality, and leads to the following equivalent block:

$$\mathbf{z}(i) := \mathbf{U}_b^{\mathcal{H}}\bar{\mathbf{z}}(i) = \bar{\mathbf{B}}\mathbf{F}_J\mathbf{s}(i) + \boldsymbol{\eta}(i), \tag{44}$$

where the noise $\boldsymbol{\eta}(i)$ is still white. Now the distance between $\mathbf{z}$ and $\mathbf{z}'$, corresponding to two different symbol blocks $\mathbf{d}$ and $\mathbf{d}'$, becomes:

$$d^2(\mathbf{z}, \mathbf{z}') = \sum_{\mu=1}^{N_t} \sum_{\nu=1}^{N_r} \|\mathbf{D}_e \mathbf{V}\mathbf{h}_{\mu\nu}\|^2. \tag{45}$$

Comparing (45) with (15), the contribution now comes from N_tN_r multipath channels. Following the same steps as in Section I, the following result can be established :

Proposition 1: *The maximum achievable diversity order is* $\sum_{\mu=1}^{N_t} \sum_{\nu=1}^{N_r} (L_{\mu\nu} + 1)$ *with* N_t *transmit- and* N_r *receive- antennas, which equals* $N_tN_r(L+1)$ *when the channel correlation has full rank.*

1. *CP-only achieves multi-antenna diversity of order* N_tN_r*;*
2. *LP-CP-only achieves the maximum diversity gain through either non-redundant but constellation-dependent, or, redundant but constellation-independent precoding;*
3. *Affine precoded CP-only and ZP-only achieve the maximum diversity gain irrespective of the underlying signal constellation.*

The linear ML processing t reach (44) requires a total of $N_d N_r = 2N_s N_r$ FFTs corresponding to each space-time coded block of (42), which amounts to $2N_r$ FFTs per information block. Channel equalization based on (44) incurs identical complexity as in single antenna transmissions. For AP-CP-only and ZP-only, the ML estimate $\hat{\mathbf{s}}_{ML} = \arg\max\_(-\|\mathbf{z} - \bar{\mathbf{B}}\mathbf{F}\mathbf{s}\|^2/N_0)$ can be obtained via exact application of Viterbi's algorithm. Relative to the two antenna case detailed in Section II-A, we can basically use the same expression for the branch metric of (39), with two modifications, namely: $r_n = [\mathbf{r}]_n$ with $\mathbf{r} = \mathbf{F}_J^{\mathcal{H}}\bar{\mathbf{B}}\mathbf{z}$, and

$$\beta_n = \sum_{\mu=1}^{N_t}\sum_{\nu=1}^{N_r}\sum_{l=0}^{L} h_{\mu\nu}^*(l) h_{\mu\nu}(n+l), \qquad \text{for} \quad n = 0, 1, \ldots, L. \tag{46}$$

We summarize the general complexity results of this section and those of Section II in the following.

Proposition 2: *The proposed space-time block coded CP-only, LP-CP-only, AP-CP-only and ZP-only systems with $N_t > 2$ ($N_t = 2$) transmit- and N_r receive- antennas require an additional complexity of $\mathcal{O}(2N_r \log_2 J)$ (respectively, $\mathcal{O}(N_r \log_2 J)$) per information symbol, relative to their counterparts with single transmit- and single receive- antenna, where J is the FFT size.*

IV. SIMULATED PERFORMANCE

In this section, we present simulation results for systems with two transmit- and one receive- antenna. For ease in FFT processing, we always choose the block size J to be a power of 2. In all figures, we define SNR as the average received symbol energy to noise ratio at the receive antenna. For reference, we also depict the (outage) probability that the channel capacity is less than the desired rate, so that reliable communication at this rate is impossible. Specifically, we calculate (28) numerically, we evaluate the outage probability at the targeted rate R as $P(C_{J\to\infty} < R)$ with Monte-Carlo simulations.

Test Case 1 (comparisons for different equalizers): We first set L = 2, and assume that the channels between each transmit and each receive antenna are i.i.d., Gaussian, with covariance matrix $I_{L+1}/(L+1)$. We investigate the performance of ZP-only with block sizes: K = 14,and P = J = 16. We adopt QPSK constellations. FIG. 6 depicts the block error rate performance corresponding to MMSE, DFE, SD, and ML equalizers. We observe that the SD equalizer indeed achieves near-ML performance, and outperforms the suboptimal block DFE as well as the block MMSE alternatives. Without channel coding, the performance of ZP-only is faraway from the outage probability at rate 2K/(K + L) =1.75 bits per channel use.

Test Case 2 (convolutionally coded ZP-only): We here use two i.i.d. taps per FIR channel, i.e., $L = 1$. We set the block sizes as $K = 127$, $P = J = 128$ for our ZP-only system, and use 8-PSK constellation. For convenience, we view each block of length $P = 128$ as one data frame, with the space time codes applied to two adjacent frames. Within each frame, the information bits are convolutionally coded (CC) with a 16-state rate 2/3 encoder. Omitting the trailing bits to terminate the CC trellis, and ignoring the rate loss induced by the CP since $L << K$, we obtain a transmission rate of 2 bits per channel use.

Turbo decoding iterations are performed. With the 16-state convolutional code, the frame error rate for ZP-only is within 2.3dB away from the outage probability.

Test Case 3 (convolutionally coded AP-CP-only over EDGE channels): We test the Typical Urban (TU) channel with a linearized GMSK transmit pulse shape, and a symbol duration $T = 3.69$ μs as in the proposed third generation TDMA cellular standard EDGE (Enhance Date Rates for GSM Evolution). The channel has order $L = 3$ and correlated taps. We use QPSK constellations, and set the block size $J = 128$. We adopt AP-CP-only that guarantees perfectly constant modulus transmissions. Within each frame of 128 symbols, the last 3 are known. Information bits are coded using a 16-state rate 1/2 convolutional code. Taking into account the known symbols, the cyclic prefix, and zero bits to terminate the CC trellis, the overall transmission rate of the proposed AP-CP-only is $(128 - 3 - 4)/(128 + 3) = 0.924$ bits per channel use, or 250.4 kbps.

As shown in FIG. 8, the system with two transmit antennas significantly outperforms its counterpart with one transmit antenna. At frame error rate of 10^{-2}, about 5dB SNR gain has been achieved. FIG. 9 depicts the performance improvement with turbo iterations, which confirms the importance of iterative over non-iterative receivers. A large portion of the performance gain is achieved within three iterations.

Various embodiments of the invention have been described. These and other embodiments are within the scope of the following claims.

CLAIMS:

1. A method comprising:

parsing a stream of information-bearing symbols to form blocks of K symbols;

precoding the symbols to form blocks having J symbols;

collecting consecutive N_s blocks;

applying a permutation matrix to the N_s blocks;

generating a space-time block coded matrix having N_t rows, each row containing N_D*J symbols;

generating N_t transmission signals from the symbols of the N_t rows; and

within N_D block transmission time intervals, communicating the N_t transmission signals through N_t transmit antennas over a wireless communication medium.

2. The method of claim 1, wherein J > K.

3. The method of claim 1, wherein J = K.

4. The method of claim 1, wherein N_t =2 and N_D =2, and applying the permutation matrix comprises applying the permutation matrix to generate the space-time coded matrix according to the following equation:

$$\begin{bmatrix} \mathbf{s}(2i) & -\mathbf{P}\mathbf{s}^*(2i+1) \\ \mathbf{s}(2i+1) & \mathbf{P}\mathbf{s}^*(2i) \end{bmatrix},$$

where *P* represents a permutation matrix, *i* represents an index into the blocks of symbols, and *s* represents symbol block.

5. The method of claim 4, wherein the permutation matrix is drawn from a set of permutation matrices $\{\mathbf{P}_J^{(n)}\}_{n=0}^{J-1}$.

6. The method of claim 4, wherein each row of a second column of the space-time block coded matrix stores a block that is a conjugated and permuted version of a corresponding block from an other row of a first column.

7. The method of claim 1, wherein precoding the symbols comprises adding a set of known symbols to each group of K symbols.

8. The method of claim 7, wherein the set of known symbols comprises a preamble and a post amble.

9. The method of claim 1, further comprising:

receiving a signal from the wireless communication medium, wherein the signal comprises a stream of received symbols;

parsing the received symbols of the input signal to form blocks of J symbols;

applying the permutation matrix to the blocks of the received symbols; and

separately demodulating transmitted data from the permutated blocks of received symbols.

10. The method of claim 9, further comprising conjugating and applying a Fast Fourier Transform (FFT) to the blocks.

11. An apparatus comprising:

an encoder to apply a permutation matrix to blocks of information bearing symbols and to generate a space-time block coded matrix of the permutated blocks of symbols;

a plurality of pulse shaping units to generate a plurality of transmission signals from the symbols of the space-time block coded matrix; and

a plurality of antennae to communicate the transmission signals through a wireless communication medium.

12. The apparatus of claim 11, wherein the encoder collects consecutive N_s blocks within a buffer, applies a permutation matrix to the N_s blocks, and forms a space-time block coded matrix having N_t rows of symbols.

13. The apparatus of claim 11, further comprising a precoder to precode the symbols to form blocks having J symbols, wherein each row of the space-time block coded matrix contains N_D*J symbols, wherein N_D represents a number of block transmission time intervals for transmitting the space-time matrix.

14. The apparatus of claim 11, wherein the precoder adds a set of known symbols to each group of K symbols.

15. The apparatus of claim 14, wherein the set of known symbols comprises a preamble and a post amble.

16. The apparatus of claim 11, wherein J > K.

17. The apparatus of claim 11, wherein J = K.

18. The apparatus of claim 11, wherein N_t =2 and the encoder applies the permutation matrix to generate the space-time coded matrix according to the following equation:

$$\begin{bmatrix} \mathbf{s}(2i) & -\mathbf{P}\mathbf{s}^*(2i+1) \\ \mathbf{s}(2i+1) & \mathbf{P}\mathbf{s}^*(2i) \end{bmatrix},$$

where P represents a permutation matrix, i represents an index into the blocks of symbols, and s represents symbol block.

19. The apparatus of claim 13, wherein the permutation matrix is drawn from a set of permutation matrices $\{\mathbf{P}_J^{(n)}\}_{n=0}^{J-1}$.

20. The apparatus of claim 11, wherein the apparatus comprises a base station within a wireless communication system.

21. The apparatus of claim 11, wherein the apparatus comprises one of a cellular phone, a personal digital assistant, a laptop computer, a desktop computer, a two-way communication device.

22. A method comprising:

applying a permutation matrix to blocks of symbols of an outbound data stream;

generating transmission signals from the permutated blocks of symbols; and

communicating the transmission signals through a wireless communication medium.

23. The method of claim 22, further comprising generating a space-time block coded matrix having N_t rows, wherein N_t represents a number of transmitters within a transmission device.

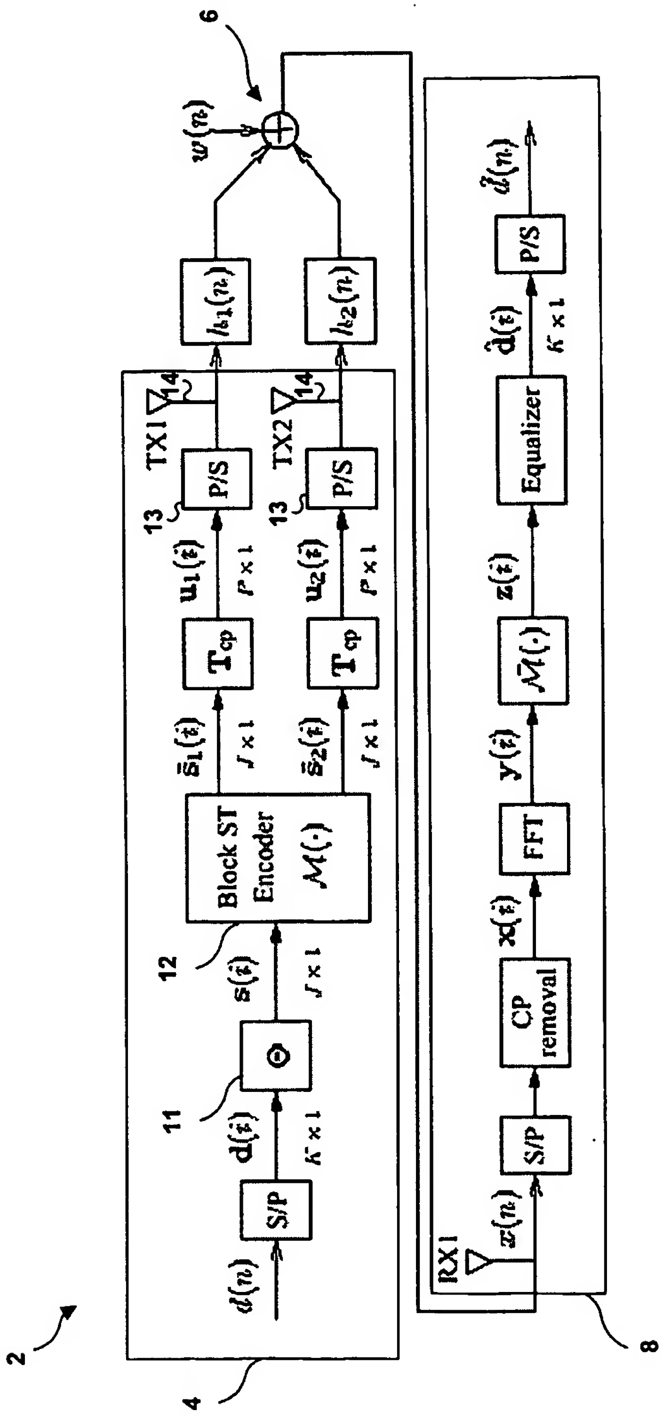
24. The method of claim 23, further comprising:

parsing the outbound data stream of information-bearing symbols to form blocks of K symbols;

precoding the symbols to form blocks having J symbols;

collecting consecutive N_s blocks; and

generating the space-time block coded matrix to have N_t rows and N_D*J symbols per row, wherein N_D represents a number of transmission time intervals for communicating the transmission signals.



2
4
6
8
11
12
13
14
S/P
Block ST Encoder
Tcp
P/S
TX1
TX2
RX1
CP removal
FFT
Equalizer
P/S
S/P

FIG. 1

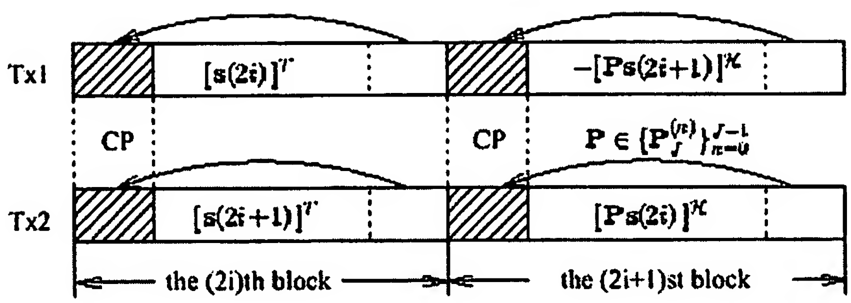

Tx1
$[s(2i)]^T$
$-[Ps(2i+1)]^{\mathcal{H}}$
CP
CP
$P \in \{P_J^{(n)}\}_{n=0}^{J-1}$
Tx2
$[s(2i+1)]^T$
$[Ps(2i)]^{\mathcal{H}}$
the (2i)th block
the (2i+1)st block

FIG. 2

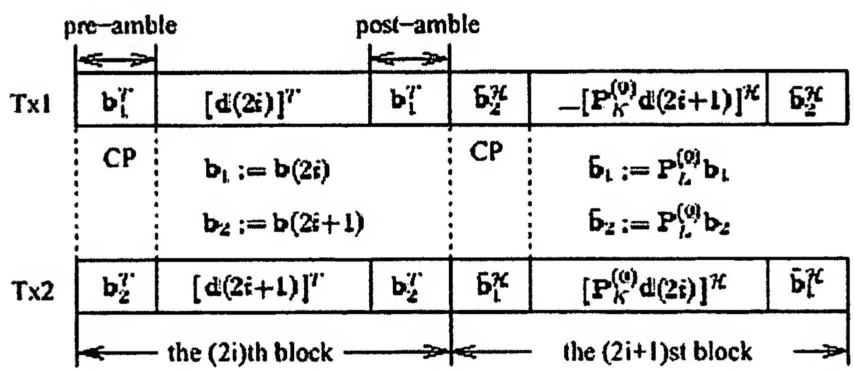

pre-amble
post-amble
Tx1
b_1^T
$[d(2i)]^T$
b_1^T
$\bar{b}_2^{\mathcal{H}}$
$-[P_K^{(0)}d(2i+1)]^{\mathcal{H}}$
$\bar{b}_2^{\mathcal{H}}$
CP
$b_1 := b(2i)$
$b_2 := b(2i+1)$
CP
$\bar{b}_1 := P_L^{(0)}b_1$
$\bar{b}_2 := P_L^{(0)}b_2$
Tx2
b_2^T
$[d(2i+1)]^T$
b_2^T
$\bar{b}_1^{\mathcal{H}}$
$[P_K^{(0)}d(2i)]^{\mathcal{H}}$
$\bar{b}_1^{\mathcal{H}}$
the (2i)th block
the (2i+1)st block

FIG. 3

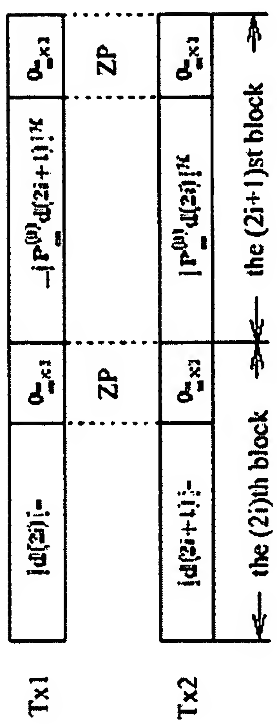

Tx1
Tx2
ZP
ZP
ZP
ZP
the (2i)th block
the (2i+1)st block

FIG. 4

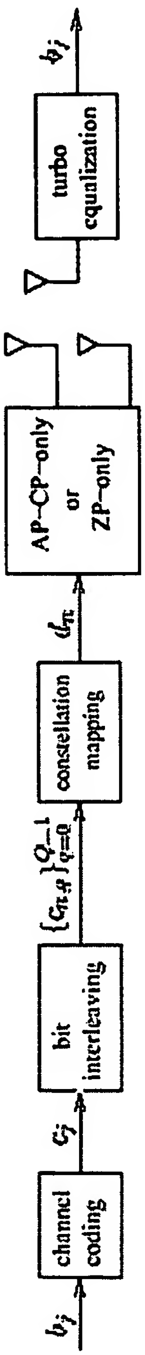
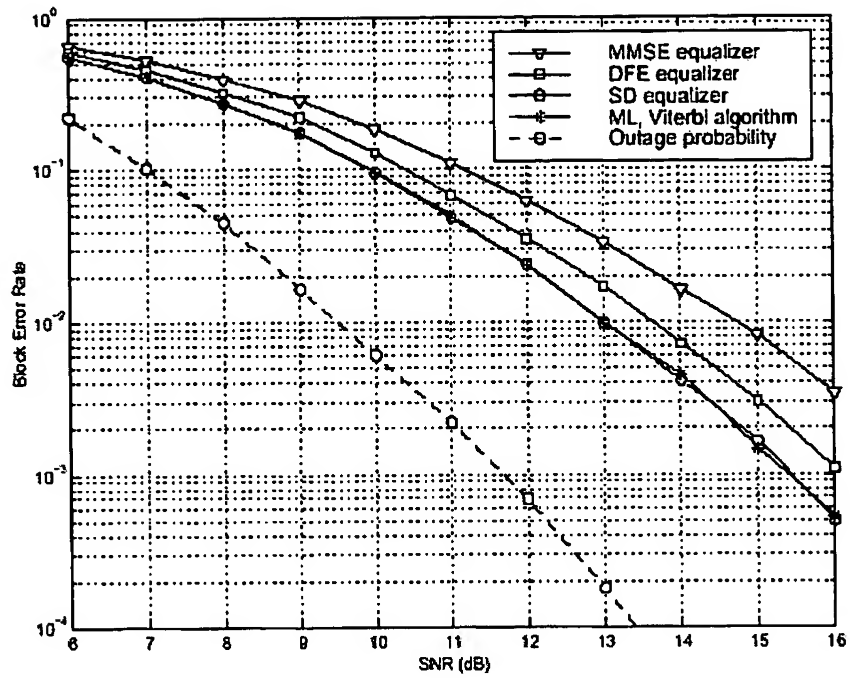
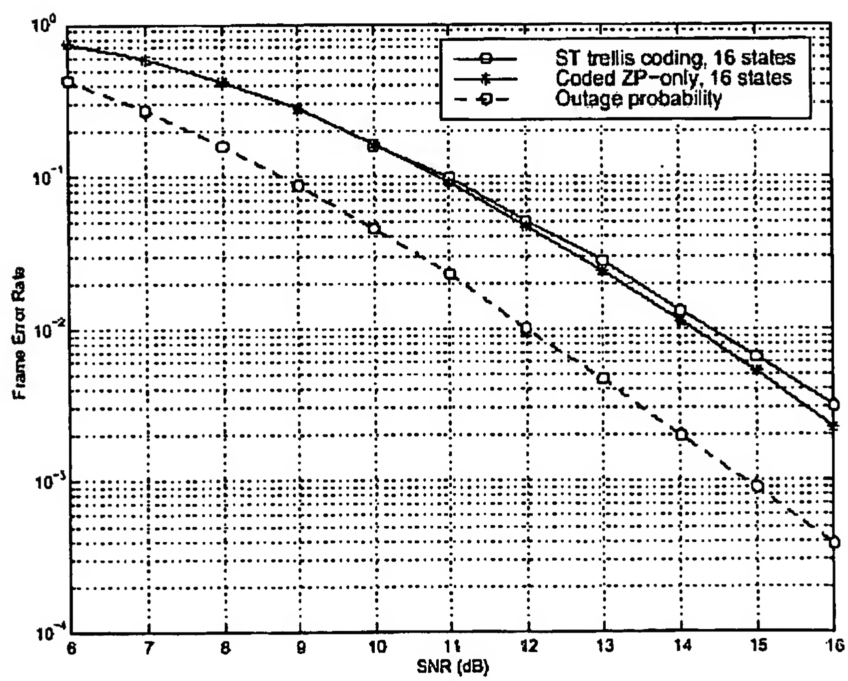

channel coding
bit interleaving
constellation mapping
AP–CP–only
or
ZP–only
turbo equalization

FIG. 5



MMSE equalizer
DFE equalizer
SD equalizer
ML, Viterbi algorithm
Outage probability
Block Error Rate
SNR (dB)

FIG. 6



ST trellis coding, 16 states
Coded ZP-only, 16 states
Outage probability
Frame Error Rate
SNR (dB)

FIG. 7

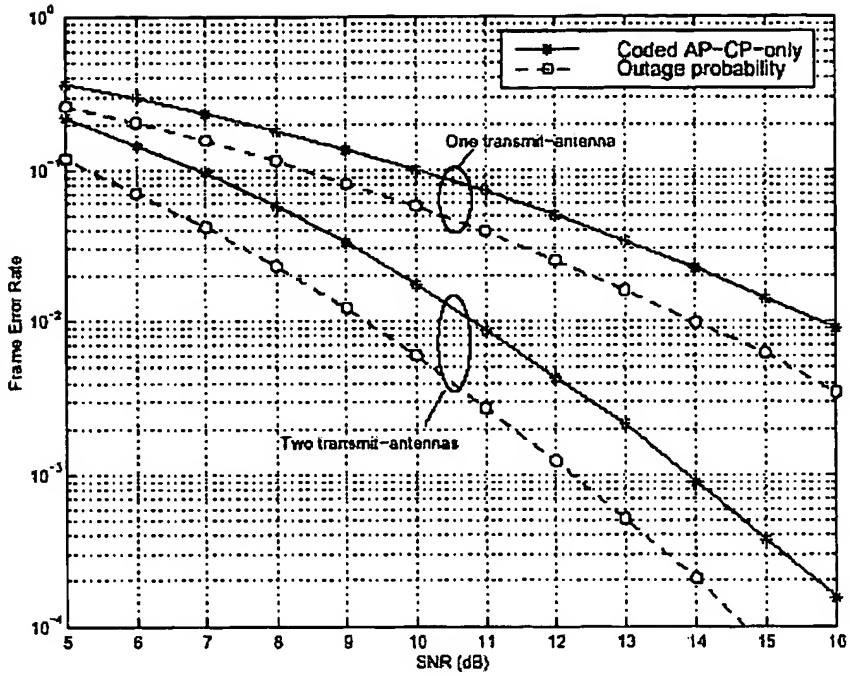

Coded AP-CP-only
Outage probability
One transmit-antenna
Two transmit-antennas
Frame Error Rate
SNR (dB)

FIG. 8

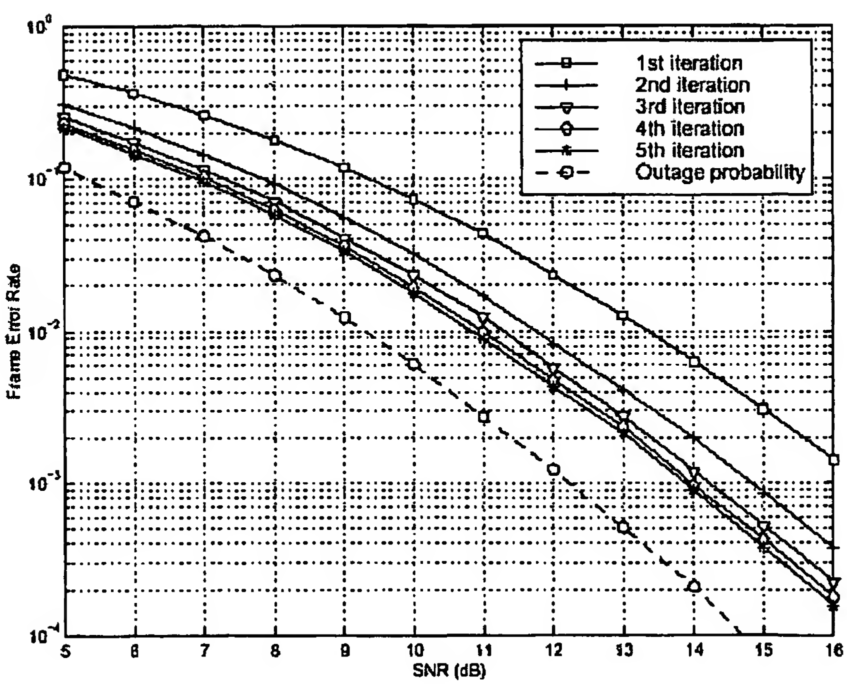

1st iteration
2nd iteration
3rd iteration
4th iteration
5th iteration
Outage probability
Frame Error Rate
SNR (dB)

FIG. 9

INTERNATIONAL SEARCH REPORT

International Application No
PCT/US 02/16897

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7 H04L1/06

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7 H04L

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

INSPEC, COMPENDEX

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	LIU Z ET AL: "Space-time coding for broadband wireless communications" WIRELESS COMMUNICATIONS AND MOBILE COMPUTING, JAN.-MARCH 2001, WILEY, UK, vol. 1, no. 1, pages 35-53, XP002210873 ISSN: 1530-8669 page 37, paragraph 2.1 page 39, paragraph 2.3.2 page 41, paragraph 3.2 -page 44 figure 5 --- -/--	1-24

[X] Further documents are listed in the continuation of box C.

[ ] Patent family members are listed in annex.

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"A" document defining the general state of the art which is not considered to be of particular relevance

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"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

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Date of the actual completion of the international search	Date of mailing of the international search report
23 August 2002	02/10/2002

Name and mailing address of the ISA	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL – 2280 HV Rijswijk Tel. (+31–70) 340–2040, Tx. 31 651 epo nl, Fax: (+31–70) 340–3016	Schiffer, A

Form PCT/ISA/210 (second sheet) (July 1992)

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category °	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	ZHENGDAO WANG ET AL: "Linearly precoded or coded OFDM against wireless channel fades?" 2001 IEEE THIRD WORKSHOP ON SIGNAL PROCESSING ADVANCES IN WIRELESS COMMUNICATIONS (SPAWC'01). WORKSHOP PROCEEDINGS (CAT. NO.01EX471), PROCEEDINGS OF SPAWC-20001. THIRD IEEE SIGNAL PROCESSING WORKSHOP ON SIGNAL PROCESSING ADVANCES IN WIRELESS COMMUNIC, pages 267-270, XP002210874 2001, Piscataway, NJ, USA, IEEE, USA ISBN: 0-7803-6720-0 the whole document -----	1-24